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ENHANCING PRE-SERVICE TEACHERS' KNOWLEDGE OF STUDENTS' ERRORS BY USING RESEARCHED-BASED CASES

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The study examined the effect of research-based cases on pre-service teachers' knowledge of students' errors on fractions. Eight pre-service teachers came from 30 participants including experienced teachers participated in cases workshops. Six research-based cases related to students' authentic errors were discussed in heterogeneous small groups. One item including three students' errors on fractions as part of the items conducted in pre- and post-test was analyzed. The result shows that more pre-service teachers became competent in identifying students' errors and offering possible pedagogical strategies corresponding to specific errors, such as, the strategy of "distinguishing the fractional part consisting of one pieces from more than one piece" for handling the errors of "ignoring the whole" and "no partitioning".

INTRODUCTION

Many studies emphasize that knowledge of students' conceptions and misconceptions is essential for teaching (Ding, 2008). The standards emphasize that errors should not be seen as dead ends but rather as springboards for inquiry (NCTM, 2000). Researchers begin to see errors as windows into student thinking (Bray, 2011). Although research has been carried out on pre-service or novice teachers' views of students' errors and responding to errors (Son & Sinclair, 2010), and on in-service teachers' handling student errors in classroom discourse (Ding, 2008), there are limited attention in enhancing pre-service teachers (PTs) knowledge of student errors.

"Failure is the mother of success". The proverb indicates that errors are often considered as catalysts for mathematics learning. Teachers hardly take advantage of mathematical errors in instruction. Many teachers tend to hide and avoid students' errors in classrooms. Handling errors in learning shows to be more effective than avoiding them, if there is clear feedback. However, prior to dealing with errors, teachers need to enhance their knowledge of students' errors, specially, identifying the sources of students' specific errors.

Teachers are learners who need to be taught in innovative ways under the use of reform-based curriculum. It is difficult for teachers to use students' errors as a teaching tool without similar learning experience in their teacher education. Fennema et al. (1996) suggest that improving knowledge of students' misconceptions can probably only acquired in the context of teaching. It implies that PTs can be learned about students' difficulties via authentic teaching or vicarious learning. However, PTs hardly have any authentic teaching experience. The research-based cases (RBC) possessing the nature of practice-based teaching can provide vicious learning, since they can foresee the complexity of the classroom.

The features of RBC used in the study include: (1) The scenarios in each case are excerpted from a real instruction; (2) Students' various correct and incorrect responses to each problem covered in each case are collected from real teaching; and (3) The part of "Discussion Questions" covered in each case are sourced from teachers' concerns being discussed in previous professional programs that we investigated.

THEORETICAL BACKGROUND

The Use of Cases

Case inquiry is an approach adopted for teacher professional development of the study. Cases provide episodes or scenarios how a teacher experienced a problem and strategies she used. Case discussion initiates reflective conversations with peers, and then fosters a collaborative disposition. Through case discussion, teachers learn to be reflective as they learn to think critically about their work and learn to see their work as problematic. As teachers talk about their work, they come to know what they know. Thus, they learn theoretical principles of mathematics and how mathematics should be taught (Merseth, 2008). Thus, the cases have the functions of collaboration, reflection, learning, and knowledge.

To achieve the ends of using cases, heterogeneous small group for discussing cases can be used for this study. Merseth (2008) report that without guidance, it is difficult for in-service teachers to have a coherent focus and promote higher thinking, let alone PTs have weak knowledge and without teaching experience. Thus, experienced teachers are invited to be the facilitators of case discussion in small groups to assist PTs becoming sensitive to students' errors.

The Importance of Knowledge of Students' Errors

Shulman (1986) has addressed the importance of handling student difficulties by putting it as a component of pedagogical content knowledge. Errors can be viewed as a diagnostic tool or as resources for promoting learning. Errors as a diagnostic tool refer to a method for detecting the causes of error. Detecting errors help to revise false knowledge, and then prevent the errors from reoccurring. Otherwise, if one's errors are not corrected immediately, the errors may repeatedly occur or bring about another related error (Brown & VanLehn, 1980).

Errors as sources for promoting learning mean errors as springboards for inquiry for stimulating student inquiry and understanding (NCTM, 2000). Rach et al. (2012) propose two ways of handling errors, outcome-oriented and process-oriented. The outcome-oriented approach proceeds directly from error detection to correction, while the process-oriented approach includes closely analyzing the errors and generating errors prevention strategies. The closer analysis contributes not only to identifying what the error is but also to knowing the sources of error. Consequently, it is more possible for a teacher to generate effective strategies to repair the errors.

Minsky's (1994) theory of negative knowledge refers to two complementary types of knowledge: negative and positive knowledge. Negative knowledge about incorrect facts and procedures is necessary to make a distinction between correct and incorrect facts and processes. Individuals are usually not taught about incorrect facts or procedures, but individuals are often staying in error situations. However, we do not warrant that all errors situations are good opportunity to learn. Hence, it is essential for individual in acquiring negative knowledge. Thus, theory of negative knowledge supports the process-oriented approach. This study takes process-oriented approach for using research-based cases in workshops. We helped PTs to identify sources of errors and figure out possible strategies for repairing errors.

Fraction requiring complex understanding is a challenge for students to learn because it defies students' intuitions from whole numbers. As a result, students have more errors on fraction than other topics. Moreover, most studies on students' errors focus on whole numbers and geometry. There is limited study on fraction. The research question of the study to be asked was: How were the elementary PTs interpret and respond to students' errors on fractions influenced by the use of research-based cases on?

RESEARCH METHOD

Participants

The 8 PTs came from 30 participants who participated in six 3-hour RBC workshops. The PTs were working on practicum in their final year of teacher preparation. The rest of 22 participants were elementary in-service teachers whose years of teaching ranged from 2 to 21. Six of the 22 teachers were master teachers for mathematics teaching.

The RBC workshops

The RBC workshops aimed at helping the participants to: (1) anticipate students' various solutions to a specific problem; (2) identify students' errors and causes of the errors; (3) discuss possible strategies the teachers could use.

Six RBCs on fractions from a casebook in which the cases we created in previous studies were employed in workshops. Case 1 demonstrates third-graders' difficulties in transforming fraction with iterating units into a par-whole model. Case 2 displays students' difficulties with the problems involving in the number of fractional part more than one. Case 3 displays fourth-graders' difficulties with renaming fraction. Case 4 displays fifth-graders' misconceptions of ordering two fractions with two dislike denominators. Case 5 illustrates fifth-graders' difficulties in equivalent fractions. Case 6 describes fifth-graders' difficulties with the distinction between 1/4 box (fraction as part-whole model) and 1/4 of a box (fraction as operator).

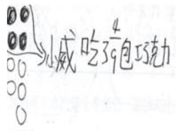
Prior to each workshop, all participants were required to read an assigned written case in advance. Each workshop began by dividing whole class into small groups with a group of 5. Each master teacher was assigned as a facilitator in each group. 8 PTs were distributed into different groups. It was followed by 1-hour group discussion on the questions listed in the "Discussion Questions". Each workshop was ended by two hours whole class discussion.

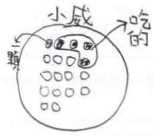
One of the authors was the leader and the facilitator of the case discussion. The leader did not provide the teachers extra information. In the whole class discussion, the participants were asked to answer the following questions: (1) What are the sources of students' errors listed in the case? (2) What evidence is there that students learned the concepts or the difficulties students have in this case? (3) What could be the possible strategies you would like to respond to each error?

Data Collections

Before taking part in the RBC workshops, we conducted a pre-test with two items including seven sub-items for all teachers. We also conducted a post-test with the same items as the pre-test at the end of the workshops three months apart from the beginning. Due to page limitation, PTs responding to item 1 as shown in Table 1 is reported here.

Item 1: Jing gave her fourth-graders to solve the problem:
A box has 18 pieces of chocolates. Joseph ate 4/9 box. How many chocolates did Joseph eat? Draw a figure to show your thought.
There were 3 pupils' errors at right column.
(1) What can be the sources of each student error?
(2) What could be the possible strategies to respond to each error?


Lee's solution


Su's solution

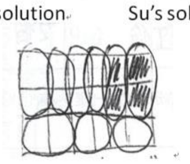

Yo's solution

Table 1: Item of pre- and post-test

Data Analysis

This study conducted cross-case analyses to examine how the PTs identify students' errors and how they would respond to the errors. Cross-case analyses were conducted to identify the similarities across cases and the differences among them, and to search for overall patterns. There were 5 codes with 8 sub-codes emerged for the sources of students' errors, and 4 codes with 9 sub-codes for possible teaching strategies for repairing the errors, the codes can be seen in Table 2 and 3.

RESULTS

PTs Enhancing Knowledge of Source of Students' Errors

Table 2 shows the overall results of their identification on the sources of errors. Due to page limitation, we will elaborate in detail in some codes.

Source of no understanding the set of objects as the whole

It can be looked into three ways on PTs' responses with the code of not understanding the set of objects as the whole. First, Lee's error was resulted from without considering a set of 18 chocolates as a unit. Second, Lee's error was resulted from the source of denominator always to be the set of the whole. Third, Lee did not consider that the number of each equal set can be more than singular entity. The PTs stated that this is a typical error when students represented a fraction in which the denominator is not equal to the number of the whole. Before the workshops, three PTs (PT5, PT6, PT8) were unable to diagnose Lee's error and no teacher focused on the code of "the number of each equal set is always one".

During the workshops, the teachers were frequently asked to distinguish the difficulties between the number of each equal-sized set is equal to one or more than one. For instance, *P1: There are 8 marbles in a bag. Joe has 3/8 box. How many marbles does Joe have? P2: There are 8 marbles in a bag. Joe has 3/4 box. How many marbles does Joe have?* The number of each set in P1 is a single entity, while that of each set in P2 consists of 3 marbles. Through case discussion, they were getting recognized the significance of relation between the number of each part and the denominator. On the post-test, the three teachers became able to identify Lee's error by using the code "the number of each equal part is just one".

Sources	Pre- test	Post-test
W:No understand the set of objects as the whole		
w1:ignoring the set of the whole	pt1, pt2,pt3	
w2:no know denominator is always the set of whole	pt4, pt7	pt1, pt2, pt7
w3:misunderstanding the concept of part should be a singular entity	--	pt3,pt4,pt5,pt6,pt8
P:No partition the set of objects into parts		
p1: no attending that 2 is the number of each part	pt1,	
p2: no attending that 9 is not the whole	pt4, pt5, pt3	pt1, pt2
p3: no partition or regrouping	--	pt3, pt4,pt5,pt6,pt7,pt8
D:Confusing the parts to be counted		
d1: misunderstanding 4 pieces as 4 equal parts	pt1, pt3, pt4, pt5,	pt1, pt3, pt4, pt5,pt6, pt8
d2: mixed up 9 equal parts with 18 pieces	pt7	pt 2, pt7
M:Misdiagnosed	pt2(2), pt5, pt6(2),	--
O:Other causes	pt6,pt7, pt8(3)	--

Table 2: PTs' identification of sources of errors in pre- and post-test.

Source of no partitioning the set of objects into parts

PTs reasoned that the causes of Sue's error in three ways : (1) Sue did not attend that 2 is the number of each set; (2) Sue was unable to identify that the denominator 9 of 4/9 is not the whole; and (3) Sue did not partition the set of chocolates into equal-sized part. The three causes were related to partitioning. As PT5 described, that Sue did not partition 18 pieces into 9 equal parts. Two pre-service teachers (PT2 and PT6) did not have correct identification of Sue's error in pre-test, but they turned out to be able to recognize the source of Sue's error in the post-test. The progress could be resulted from the discussion in Case 5 workshop. The PTs were asked to discuss the problem 3*: "A box contains 24 cans of drinks. How many boxes are 18 cans?(1) Joe packs them into small bags. Each bag has 3 cans. How many boxes are 18 cans?(2) Chris divides the cans of drinks into 4 small bags. How many boxes are 18 cans?"*

Source of confusing the parts to be counted

In fraction, the numerator tells how many parts. PTs anticipated the sources of Yo's error in two ways: confusing 9 equal parts with 18 pieces and misunderstanding 4 pieces as 4 equal parts. The latter was the major source identified by the PTs (PT1, PT3, PT4, PT5). PT8 did not accept the use of continuous model for the discrete objects as the source of Yo's error in pre-test. Her pedagogical concept was revised through case discussion. In Case 1 workshop, we discussed that discrete objects is not necessarily to be represented as discrete model, discrete objects can be represented by a continuous model, such as Yo's. In the post-test, PT8's anticipation of the source of Yo's error was not on the model, rather changed to the source of misunderstanding 4 pieces as 4 equivalent sets.

PTs Enhancing Knowledge of Possible Pedagogical Strategies Handling Errors

PTs wrote 9 teaching strategies against students' errors on item 1. Table 3 shows the overall results with this matter.

Sources	Pre-	Post-
To understand the set of objects as the whole		
T1: draw out the whole	pt1, pt2,pt3	pt1, pt2,
T2: give more examples	pt4,	pt7
T3: to distinguish the fractional part consisting of one pieces from more than one piece	--	pt3,pt4,pt5,pt6,pt8
To partition equally the set of objects into parts		
T4: start from 1/9 box and move forward to 4/9 box.	pt3	pt3,
T5: to partition or regroup	pt1,pt5	pt2. pt7
T6:to distinguish the fractional part consisting of one pieces from more than one piece	--	pt1, pt4, pt5, pt6, pt8
To understand the parts tell how many to be counted	--	
T7: start from 1/9 box and move forward to 4/9 box.	pt1, pt3, pt4,	pt1, pt3
T8: distinguish 4/9 box from 4/18 box	pt2,pt5, pt7	pt2,pt5, pt7,pt8
T9: to designate what is one part		pt6
M:Conginitive conflict	pt4,pt7(2)	pt4
O:Others	pt2,pt5,pt6(3),pt8(3)	

Table 3: PTs' possible pedagogical strategies responding to errors in pre- and post-test.

Strategies for constructing the concept of the set of objects as one whole unit

In pre-test, the possible teaching strategies repairing Lee's error included: (1) by asking to draw out the one whole unit; (2) by providing more examples; and (3) by distinguishing a fractional part in an example with one piece from another example with more than one pieces. Prior to case workshop, most of the PTs mentioned that 18 chocolates as one whole unit must become a conceptual entity, so that their responses to Lee's errors involved "showing/drawing/telling" Lee one whole unit. After cases workshops, the most prominent category PTs responding to Lee's error were by distinguishing a 4/18 of 18 chocolates from 4/9 of 18 chocolates. Making students solve more examples or counterexamples to compare for causing Lee's cognitive conflict was frequently mentioned by T4 and T7. They were cautious that the concept of a whole underlies the concept of a fraction. A whole is treated as a unit.

Strategies for partitioning equally the set of objects into parts

In pre- and post-test, PTs consistently agreed that partitioning is a fundamental to an understanding of factions. They would use three possible teaching strategies to help Sue to partition a set of objects into equal parts. First, they would start with 1/9 box to partition 18 chocolates into 9 equal sets and continually move forward to 4/9 box. Second, the PTs would provide Sue more examples of partitioning. Third, they would help Sue to understand that in discrete model the fractional part consists of two pieces in this instance but may be one piece in others. The third strategy for dealing with Lee's and Sue's errors have never proposed by the PTs in the pre-test. Through cases workshop, as problem 3 described previously, this strategy became most popular when the PTs helped Sue to construct the concept of whole-unit and partition, especially for PT4, PT5, PT6, and PT8.

Strategies for understanding the parts tell how many to be counted

In discrete model of fractions, even though understanding the two fundamental concepts including unit-whole and partitioning, it is possible to have difficulty in determining the fractional parts corresponding to the numerator of symbolism. Regarding this error, PTs would take two pedagogical strategies to handle. First, starting from 1/9 box and gradually moving forward to 4/9 box. Second, distinguishing 4/9 box from 4/18 box. As T1 described that "*I would ask Yo to figure out the number of chocolates in 1/9 box, then stepped forward to find out the number of chocolates in 4/9 box. Moreover, the language "part" cannot be ignored*". In the pre- and post-test, three teachers (PT2, PT5, PT7) coherently used this strategy for making a distinction between 4/9 box and 4/18 box by drawing a picture.

DISCUSSION

This study attributed enhancing PTs' knowledge of students' errors on fractions to the use of research-based cases. The following factors were analysed. (1) Students' errors in the written cases were authentic rather than imagined. These cases helped PTs to foresee the errors patterns students made. Such knowledge would be useful for PTs to anticipate what errors students would have in related mathematics topics. (2) Experienced teachers worked with the PTs in case discussion. Each experience teacher in each group played a role of abler in case discussion. Case discussion initiated social interaction. Hence, knowledge of students' errors was improved.

The sources of errors PTs identified were almost related to epistemological causes which refer to the nature of mathematics concepts. This finding was not consistent with previous studies that teachers tend to attribute students' errors to psychological cause (Bingobali et al., 2011). The difference of the study from previous study was in that the task used in the study was involved in students' actual errors instead of by asking superficial questions, such as “what can be the causes of the students’ error in learning a mathematical concept?”.

Various possible teaching strategies for repairing students’ errors were mentioned by the PTs. Such strategies would be more efficient than the strategy of “clear explanation and description” mentioned in previous studies (Son & Sinclair, 2010). Since such strategies do deal with the sources of accurate students’ errors, this makes it possible for the PTs to anticipate exactly the sources of students’ errors and to use acceptable teaching strategies for repairing errors in the future. The authentic students’ errors as the task used in the study became the feature of the study. This study also provided closer analysis of students’ errors. The Rach et al.’s (2012) process-oriented approach suggests that it is more possible to generate effective strategies to repair the errors. Whether the possible pedagogical strategies through closer analysis for handling students’ errors on fractions are efficient, it should be further examined in classroom teaching. The issue becomes a research question for future study.

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