Lin, P. J. (2012). Enhancing teachers' knowledge of students learning by using research-based cases. *Paper presented at the 92nd annual meeting conference of Association of Teacher Educators (ATE).* Feb. 10-15, San Antonia, Texas.

Enhancing Teachers' Knowledge of Students Learning by Using Research-Based Cases

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ABSTRCT

The study intended to use research-based cases to enhance teacher knowledge of students' learning of mathematics. Teacher knowledge of students' learning focused on anticipating students' various solutions and arranging the order of students' various solutions. There were five teachers participated in the case discussion groups. The decimals cases covered in a casebook were the intervention of the professional development program. The pretest and posttest were conducted for the participants. The result indicates that anticipating, categorizing, and sequencing students' various solutions are the indicators of teachers' knowledge of students' learning. Teachers' mstering in predicting students' anticipated solutions is a strategy for improving teachers' understanding students' prior knowledge Ordering students various solutions for discussion was based on the degree of difficulty of problems.

INTRODUCATION

The innovative curriculum is aimed at developing students' abilities to explore, communicate, conjecture, and reason logically (MET, 2001). The change of mathematics teaching switches from traditional instruction approach to contemporary view of mathematics teaching. The contemporary view of mathematics teaching emphasizes conceptual understanding and responding to individual students' experiences and needs instead of treating all students alike. The reform documents including textbook define new roles for teachings related to the issues of knowledge constructing and supportive scaffolding. Thus, many inservice teachers are challenged by the discrepancy between the reform version of mathematics instruction and their own teaching.

Helping teachers toward a contemporary view of teaching seems to require new experience. One of the ways to acquire the new experience is based on others' experiences. The use of cases that reflect aspects of classroom practice is one way to learn from others' real experience. Cases can be performed with different purposes (Merseth, 1996). Cases can be exemplars to establish the best practice or to make the effective teaching more public. The exemplary cases aim to assist teachers to develop the skill of critical reflection on their own practice (Kleinfeld, 1992; Lin, 2002; 2005; Stein, et al., 2000). These studies show that cases help teachers becoming more reflective practitioners, since cases reflect real situations and create challenges for teachers (Barnett, 1998). Thus, the use of written cases in a compiled book (Lin, 2007) that is collected from others' real experience in teaching developed from a longitudinal teacher professional development program. Thus, the written cases used in the study are based on research result. The written cases were constructed by a professional team consisting of 6-8 teachers and teacher educators.

Knowledge of students' learning has been activated since from Cognitive Guided Instruction. Afterwards, it is followed by many researchers, such as in Deborah Ball's research team, they suggest that knowledge of content and students (KCS) is one of six domains of mathematics for teaching (MKT) (Hill, Ball, & Schilling, 2008). KCS is acted as content knowledge intertwined with knowledge of how students think about, know or learn this particular content. This indicates that knowledge of students learning is essential for mathematics teachers. Knowledge of students learning refers to knowing students' prior knowledge and experience, cognitive development, ways of thinking, ways of learning, anticipation of students' solution, identification of students' various solutions, interpretation of students' solutions, ordering students' various solution s for discussion in public. A great number of studies indicate the effect of knowledge of students learning on mathematics teaching and on students' learning (e.g., Hill, Ball, & Schilling, 2008). However, there is little research on how to increase teacher's knowledge of identifying students' various solutions, anticipating students' various solutions, and selecting and ordering students' various solutions for discussion in public. Thus, the study is intended to increase teachers' knowledge of students' learning through the use of research-based written cases of mathematics teaching.

THEORETICAL FRAMEWORK OF THE STUDY

The theoretical framework of developing a teacher professional development was based on the rationale of a teacher as a learner. Thus, the theoretical framework consists of three components: learning goals, learning strategies and learning contexts. The learning goals was intended to achieve teachers' three main knowledge domains: mathematics content knowledge, pedagogical content knowledge, and knowledge of students' learning, in particular, knowledge of student learning. Critical reflection was also considered as a kernel part of the learning goals.

Learning strategy for the teachers is the use of written cases. It consists of reading the cases and case discussion. The classroom setting and professional dialogue meetings for the collaborative professional team are two social contexts for teachers' mutual support. These two create the opportunity for teachers to become reflective practitioners. Classroom observations created opportunities for communication what teachers learning from classroom. Routine weekly meetings create the opportunities for teachers making reflection on their previous instruction and for their future instruction. it creates the possibility of initiating cognitive conflict with their previous cognition. Thus, social interaction, cognitive conflict, and reflection were the mechanisms of initiating teachers learn to teach.

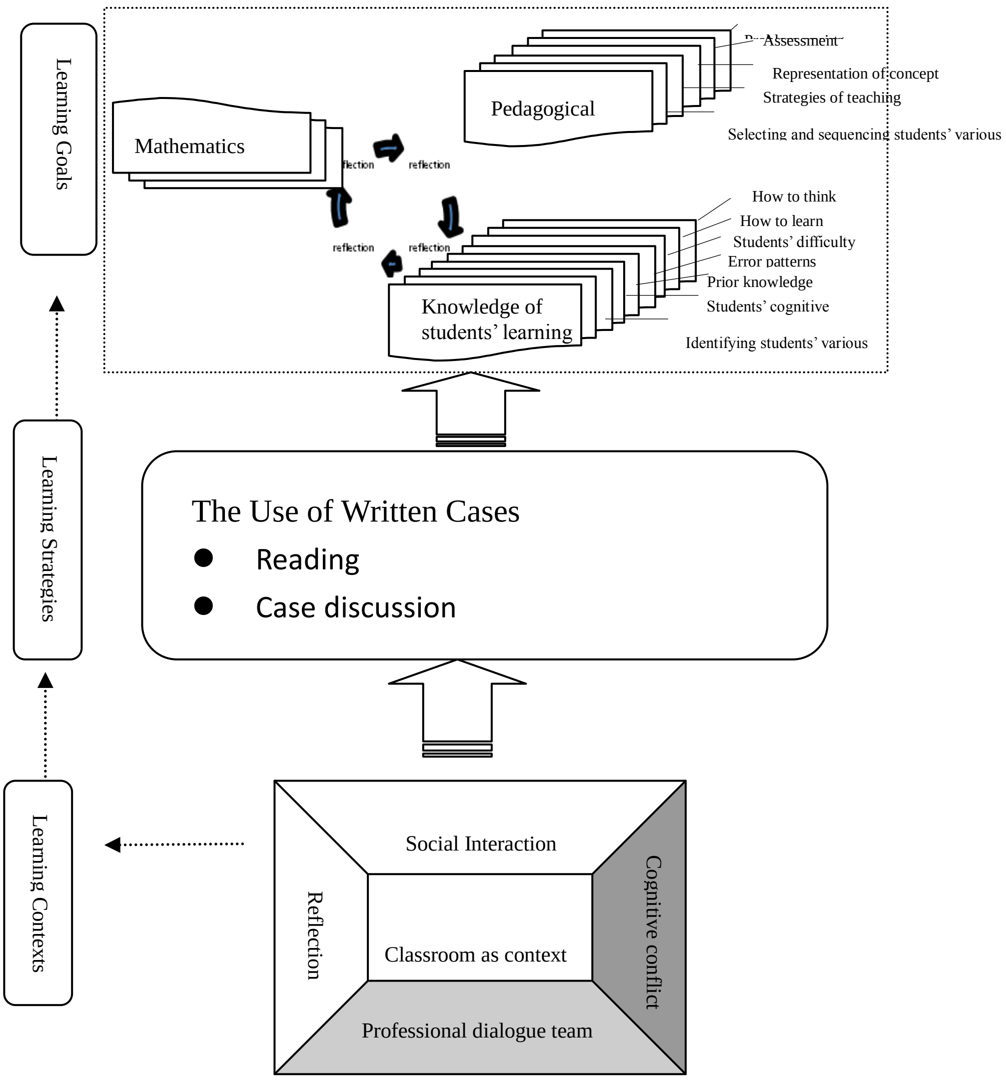

Figure 1: A theoretical framework of the study

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RESEARCH METHOD

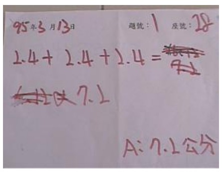
Participants

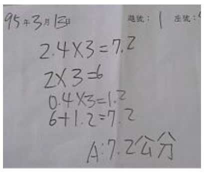
A school-based professional team consisting of six teachers was set up for discussing the written cases. T0, one of the teachers as a facilitator and the five teachers, T1, T2, T3, T4, and T5 as participants participated in the study. The five female teachers' teaching years were 6, 15, 23, 6, and 10, respectively. Excepting T5 with mathematics major background, others are not major in mathematics. The facilitator with ten years of teaching has a master degree of mathematics education. She has been received a 30-hours training for skilling in the use of the cases. The facilitator was trained to be played different roles in facilitating, probing and giving feedback to teachers and create the opportunities for the participants to sit together to discuss the written cases.

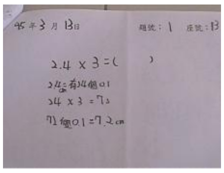
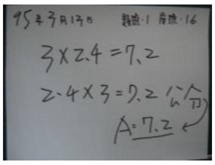
Written Cases and Case Discussion

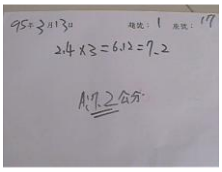
A case book of decimals is composed of six written cases of decimals teaching. Each written case consists of seven components: background of the case, instructional objectives, target grade, prior knowledge, flow of teaching including students-teacher dialogues, questions for discussion, and teachers' guides for using the case. The students' various solutions and question for discussion covered in a written case was listed below.

Table 1：The example of Students' various solutions and question for discussion covered in a written case

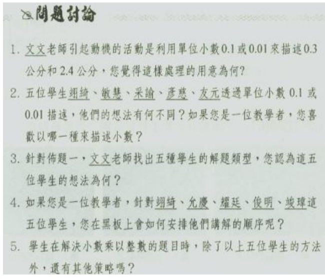
<table>
<tr><th>Components</th><th>Content</th></tr>
<tr><td>Problem to be solved</td><td>Each paperclip has 2.4 cm in length. How long are 3 paperclips altogether?</td></tr>
<tr><td>Students' various solutions</td><td>


Yu-Chi


Yuan-Ching

Yiao-Ting


Jian-Ming


Jian-Wei
</td></tr>
</table>

<table>
<tr>
<td>Question for discussion</td>
<td>
問題討論
1. 文文老師引起動機的活動是利用單位小數0.1或0.01來描述0.3公分和2.4公分，您覺得這樣處理的用意為何?
2. 五位學生翊綺、敏慧、采諭、彥慈、友元透過單位小數0.1或0.01描述，他們的想法有何不同？如果您是一位教學者，您喜歡以哪一種來描述小數？
3. 針對佈題一，文文老師找出五種學生的解題類型，您認為這五位學生的想法為何？
4. 如果您是一位教學者，針對翊綺、允慶、耀廷、俊明、竣瑋這五位學生，您在黑板上會如何安排他們講解的順序呢？
5. 學生在解決小數乘以整數的題目時，除了以上五位學生的方法外，還有其他策略嗎?</td>
<td>English version:
1. The instructor initiated by reviewing the units of 0.1 and 0.01 to describe 0.3cm and 2.4cm for students, What is his purpose?
2. The five students describe a decimal number by using 0.1 or 0.01. Which of the solution would you like to use for describing a decimal number?
3. What the differences among the five students' solutions?
4. How would you like to order the five solutions for discussion in classroom? Why?
5. What else of solutions would you like to predict?</td>
</tr>
</table>

Sources：Lin(2007), p.69～70

Each case was initiated by reading each written case individually. It is immediately followed by the case discussion. The discussion session was attempted to encourage the participants to answer the question in the Question for Discussion covered in the written cases. There were seven meetings for the case discussions in which the written cases focus on interpreting, predicating, and sequencing students' various solutions. The number of questions covered in the case book corresponding to predicting, categorizing, interpreting, and ordering students' various solutions is listed in Table 2.

Table 2: The number of questions covered in the case book corresponding to predicting, categorizing, interpreting, and ordering students' various solutions.

Characteristic / # of questions / Case #	Predicting	Categorizing	Interpreting	Ordering
	students' various solutions			
Case 1			1	
Case 2			1	
Case 3	1			1
Case 4		1	1	1
Case 5	2	5	5	1
Case 6		2	4	2

Data Collection

To examine the effect of the use of cases on teachers' knowledge of students' learning, the instruments for assessing teachers' knowledge of students learning were developed for pre- and post-test. To increase the reliability and validity for the study, the cross table of item analysis were used. There were six items in the pretest and posttest instrument, respectively. The difficulty of pre-test and post-test keeps the same level by considering the numerals and context of the problems. The example of pretest and posttest for assessing teachers' skills in predicting students' anticipated solution, selecting and ordering students' various solutions for discussion in public were contrasted as in Table 3.

Table 3: Examples of pretest vs. posttest for assessing teachers' knowledge of predicting and ordering students' various solution for discussion in public

<table>
<tr><th>Test
Characteristics</th><th>Pre-test</th><th>Post-test</th><th>Item #</th></tr>
<tr><td>Predicting students' solutions</td><td>A strip has 0.3 m. How long are 3 strips in total? What would be students' possible solutions?</td><td>A strip has 0.2 m. How long are 3 strips in total? What would be students' possible solutions?</td><td>1 & 3</td></tr>
<tr><td>Categorizing students' multiple solutions

Ordering students' various solution for discussion in public</td><td>A box has 10 egg cakes. David ate 0.3 box per day. How many cakes were eaten for 3 days? Students' possible solutions were as follows.
<u>Method 1:</u>
o.3 box=3 cakes;3x3=9 cakes.
A: 9 cakes.
<u>Method 2</u>:
0.3x3=0.9; 0.3+0.3+0,3=0.9
A: 0.9 box.
<u>Method 3</u>:
0.3x3=0.9<table><tr><td>●</td><td>●</td><td>●</td><td>●</td><td></td></tr><tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr></table>A: 9 cakes.
<u>Method 4</u>:
3/10=0.3; 0.3x3=0,9=9/10<table><tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td>●</td><td>●</td><td>●</td><td></td></tr></table>A: 9 cakes.
<u>Method 5</u>:
0.3x3=0.9; A: 0.9 cakes.
<u>Method 6</u>:
3x3=9; 0.1x9=0.9; A: 0.9 box.
(1) How would you like to categorize them?
(2) Which of the methods is the most frequent use for students?
(3) If you were the instructor, how would you arrange the order of the various solutions? Show your reason.</td><td>A box has 10 egg cakes. David ate 0.2 box per day. How many cakes were eaten for 3 days? Students' possible solutions were as follows.
<u>Method 1:</u>
o.2 box=2 cakes;2x3=6 cakes.
A: 6 cakes.
<u>Method 2</u>:
0.3x3=0.6; 0.2+0.2+0,2=0.6
A: 0.6 box.
<u>Method 3</u>:
0.2x3=0.6<table><tr><td>●</td><td></td><td></td><td></td><td></td></tr><tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr></table>A: 5 cakes.
<u>Method 4</u>:
2/10=0.2; 0.2x3=0,6=6/10<table><tr><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>●</td><td></td><td></td><td></td><td></td></tr></table>A: 6 cakes.
<u>Method 5</u>:
0.2x3=0.6; A: 0.6 cakes.
<u>Method 6</u>:
2x3=6; 0.1x6=0.6; A: 0.6 box.
(1) How would you like to categorize them?
(2) Which of the methods is the most frequent use for students?
(3) If you were the instructor, how would you arrange the order of the various solutions? Show your reason.</td><td>4 &5

2 &6</td></tr>
</table>

Data Analysis

The data collected were to contrast and compare the difference between the pretest and posttest of teachers' performance in predicting students' various solutions and ordering students' various solutions for solving the problems of decimal x positive integer and positive integer × decimal.

RESEARCH RESULTS

In accordance with the posttest comparing to pretest, it is found that the use of cases contributes to supporting teachers in predicting students' anticipated solutions appropriately. Here appropriate predication means that the teachers' predictions were based on students' prior knowledge and learning sequence.

The Improvement of Teachers' Knowledge of Anticipating Students' Solutions

Improving teachers' knowledge of anticipating students' various solutions in solving the problems of decimal number × whole number

In the pretest, there were five methods were predicted by the teachers for students to be used in solving the problems of decimal number x whole number. The five anticipated methods: 1) repeated addition, 2) converting decimal number to fraction, 3) mutual conversion of natural units (such as 1 m converted to 100cm), and 4) missing number in proportion (e.g., a:b=c:d). The method of missing number in proportion is beyond students' prior experience, while the other four methods are matched to students' prior knowledge. The method of 0.1 or 0.01 as iterated unit was not used by anyone of the teachers involving in the pretest.

Table 4: Comparisons of pretest and posttest for teachers predicting students' various solutions in solving the problems of decimal number × whole number

Pretest: A strip has 0.3 m. How long are 3 strips in total? What would be students' possible solutions? Posttest: A strip has 0.2 m. How long are 3 strips in total? What would be students' possible solutions?				
	Pretest		Posttest	
Methods	Solutions	Teachers	Solutions	Teachers
Repeated addition	0.3+0.3+0.3=0.9	T5，T4，T3，T2，T1	0.2+0.2+0.2=0.6	T5，T4，T3，T2，T1
Converting decimal number to fraction	$\frac{3}{10}\times\mathbf{3}=\frac{9}{10}=0.9$	T5， T3， T1	$\frac{2}{10}\times\mathbf{3}=\frac{6}{10}$	T5， T3， T1
0.1 as a counting unit	0.3×3＝three 0.1 repeated 3 times= 0.1repeated 9 times＝0.9	---	0.2×3＝three 0.1 repeated 2 times= 0.1repeated 9 times＝0.9	T5，T2
Mutual conversion of natural units	0.3m＝30cm 30×3＝90 90cm＝0.9m	T5	0.2m＝20cm 20×3＝60 60cm＝0.6m	T5，T1
Missing number in proportion	1：0.3＝3：□ □＝0.3×3÷1＝0.9	T5，T1	1：0.2＝3：□ □＝0.2×3÷1＝0.6	---

The method of 0.1 or 0.01 as iterated unit was listed in the written case for case discussion. For example, Yiao-Ting's solutions in the written case was: 2.4x3=( ), 2.4 are viewed as 0.1 iterated 24 times. Then, 24x3=72, 0.1 iterated 72 times is 7.2 Students' prior knowledge is given in the written case, such that the teachers realized the method of missing number in proportion learned at sixth grade. Table 4 displays the difference between pretest and posttest for the teachers predicting students' various solutions in the problem of decimals × whole number.

Through the use of cases, the teachers improved their knowledge in predicting students' various solution. In the case discussion, they all agreed that the proportional problems were not learned until students are up to sixth grade. In the posttest, the method of missing number in proportion beyond students' prior knowledge was no more on the list of the teachers' prediction. Moreover, more students' possible appropriate solutions were predicated by each teacher, as seen in Table 4.

In the posttest, each method to be used by students was predicted by more teachers. For instance, each teacher did not anticipate the method of 0.1 as iterated unit in the pretest, while two teachers, T5 and T2, anticipated it in the posttest.

In the pretest, T2 did not recognize that students solving the problems of whole number × decimals are more difficult than those of decimals × whole numbers. As a result, 0.3×3=3×0.3=0.9 was predicted by T2 in the pretest. Through the case discussion, T2 did not predict students solving the problem by using 0.2x3=3x0.2=0.6 in the posttest.

Although the method of mutual conversion of natural units was not covered in the written case, merely T5 predicted in the pretest that students would use this method; 0.3m is first converted into 30cm; then, 0.3 m×3 is transformed into 30cm×3, as the multiplication of two whole numbers. In the posttest, one more teacher, T1 used the method of mutual conversion of natural units as one of students' possible solutions.

Improving teachers' knowledge in anticipating students' various solutions in solving the problems of whole number × decimal number

It is found that teachers had a common predication on the method of converting decimal number to fraction for students solving the problems of whole number × decimals and decimals × whole number. There were only two teachers perceived the decimals × whole number as prior experience of whole number × decimals. Thus, converting whole number × decimals into decimals × whole number becomes as one of the method for some teachers predicating the students solving the problems of whole number × decimals.

Table 5: Comparisons of pretest and posttest for teacher predicting students' various solutions in solving the problems of whole number x decimal number

Pretest：There were 6 cakes in a box. David ate 0.3 of the box. How many of the cakes were eaten? What kind of solutions would students come up for solving this problem?

Posttest：”There were 10 cakes in a box. David ate 0.4 of the box. How many of the cakes were eaten? What kind of solutions would students come up for soling this problem?

	Pretest	Posttest

Methods	Solutions	Teachers	Solutions	Teachers
Convert to decimal x whole number	10×0.3＝ 0.3×10	T5， T3	10×0.4＝ 0.4×10	T5， T4，T3，T2
Convert to fraction	$\frac{1}{10}$box＝1cake 0.3box＝$\frac{3}{10}$box ＝3cakes	T5， T4，T2，T1	$\frac{1}{10}$box＝1 cake 0.4box＝$\frac{4}{10}$box＝ 4cakes	T5， T4，T2，T1
0.1as a counting unit	10×0.3＝ 10×(three 0.1)＝ thirty 0.1＝3	---	10×0.4＝ 10×(four 0.1)＝ forty 0.1＝4	T5，T2
propotion	1：10＝0.3：□ □＝10×0.3÷1＝3	T5	1：10＝0.4：□ □＝10×0.4÷1＝4	---
Enlarge and reduce	10×3＝30 （enlarge 10 times），then reduce 10 times	---	10×4＝40（enlarge 10 times），then reduce 10 times	T5
Convert to multiples of 0.1	10×0.3＝ （10×0.1box)× 3 ＝1cake×3＝3	T2， T1	10×0.4＝ （10×0.1box)× 4 ＝ 1cake×4＝4	T4， T3，T2，T1

Table 5 displays the difference between pretest and posttest for each teacher predicting students' various solutions in the problems of decimal multiplication.

Table 5 shows that there were 6 solutions teachers used to predict students' solutions for solving the problems of whole number × decimals. Two teachers, T3 and T4 predicted students' unique solution in the pretest, while they predicted more students' solutions in the posttest. Although the method of enlarging and reducing were demonstrated in the written case, it is not in the list of teachers' prediction. This indicates that through the case discussion, teachers' prediction did not focus on more solutions, rather, on students' prior knowledge. We also found that T5 with mathematics education background have more predictions both in pretest and posttest than other teachers.

Predicting students' anticipated solutions as a strategy for understanding their prior knowledge

It is found that the skills in predicting students' anticipated solutions is a strategy for improving teachers' understanding students' prior knowledge, since each anticipated solution is based on students' prior knowledge. Otherwise, it is not an appropriate predication. Table 6 depicts each teacher's prediction to each method comparing pretest to posttest.

Table 6: Frequencies of each teacher's prediction to each method for pretest and posttest.

	Methods	T1		T2		T3		T4		T5	
		Pre-	Post	Pre-	Post	Pre-	Post	Pre-	Post	Pre-	Post
Decim	Repeated addition	❍	❍	❍	❍	❍	❍	❍	❍	❍	❍
	Converting decimal	❍	❍			❍	❍			❍	❍

	number to fraction										
	0.1 as iterated unit				❍						❍
	Mutual conversion of natural units		❍							❍	❍
	missing number in proportion	❍								❍	
Whole number x decimals	Converting into Decimal x whole number				❍	❍	❍		❍	❍	❍
	Converting decimal number to fraction	❍	❍	❍	❍			❍	❍	❍	❍
	0.1 as iterated unit				❍						❍
	missing number in proportion									❍	
	Enlarge and reduce										❍
	Converting decimals into integer times 0.1	❍	❍	❍	❍		❍		❍		
Total		5	5	3	6	3	4	2	4	7	8

The method of missing number in proportion with 3 frequencies predicted by teachers in pretest, but it was removed off in the posttest. The number of possible solutions to be used by students solving the two problems the teachers provided in the pretest was 20 frequencies in total. Each teacher averagely predicted 2 students' solutions for each problem in pretest. The knowledge of teachers' predicting students' solutions was increased in the posttest, because there were 27 students' solutions predicted by 5 teachers. Each teacher predicted 2.7 students' solutions.

According to Table 6, teachers would not use a same method to predict different problems. For instance, the multiplication of fraction is fifth-graders' prior knowledge, so that the method of converting decimal to fraction were predicted by several teachers. Comparing the pretest and posttest, T5 and T1, two of the five teachers coherently predicted the method of converting decimal to fraction used by students solving the problems of the multiplication of decimals.

The Improvement of Teachers' Knowledge of Sequencing Students' Various Solutions for Discussion in Public

Degree of difficulty as the basis of sequencing students' various solutions for discussion

Ordering students various for discussion was based on the degree of difficulty of problems. Due to lacking of good understanding on students learning in decimal, teachers had a difficulty with the interpretation students' various solutions. As a consequence, teachers were not equipped with ordering students' multiple solutions. For instance, method 1 is the easiest method, whereas only T1 put it in the first. On the contrary, all teachers did not sequence the most difficult method, method 6, in the final order for discussion.

During the time of between pretest and posttest, teachers have been discussed 28 questions about categorization and sequences students' various solutions covered in the Discussion Questions of the written cases. This is potential to contribute to teachers' understanding about the sequence of the students' prior knowledge and students' understanding. For instance, all teachers have consistent agreement with "0.1 as a counting unit" as the most difficult method, seen as Table 7. Thus, they all agreed that Yiao-ting should be invited at last to explain his method, as indicated in Table 1.

Table 7: Comparisons of pretest and posttest for teacher ordering students' various solutions for discussion

	T1	T2	T3	T4	T5
Pretest	①→⑥→③④→②→⑤	⑥→②→⑤→③→④→①	④→①→③②⑤	④→①③→②⑥→⑤	③④→①→②⑤→⑥
Posttest	①→②③④⑤→⑥	④→②→③→⑤→①→⑥	①→⑥→③→④→②→⑤	①→③→④→②⑤⑥	①→④→②③⑤→⑥

Besides, all teachers arranged the easiest method, repeated addition, as the first order to be reported, as Yu-chi shown in Table 1. Yuan-ching decomposed 2.4 into 2 plus 0.4. The method is more difficult than Yu-chi's and Yiao-ting's. Thus, the reasonable sequence arranged to be discussed in public is from Yu-chi's solution to Yiao-ting's solution, and then Yuan-ching's solution, as seen in Table 1. Nevertheless, four teachers (excepting T4) sequenced in the order of Yu-chi→Yuan-ching→Yiao-ting, as seen in Table 8. Until the case discussion, the facilitator T0 invited T2 to illustrate Yiao-ting's solution by using the method of "0.1 as a counting unit". In the posttest, more teachers (T1, T2, T4, T5) put the method 6 (i.e., missing number in proportion) in correct order for discussing in public, as seen in Table 7.

Table 8: Teachers ordering the sequence of students' various solutions shown in written cases indicated as Table 1.

Teachers	T1	T2	T3	T4	T5
Case discussion	Yu-chi→Yuan-ching→Jian-wei, Jian-ming→Yian-ting	Yu-chi→Jian-ming→Yuan-ching→Jian-wei→Yian-ting	Yu-chi→Yuan-ching→Jian-wei, Jian-ming→Yian-ting	Yu-chi→Yian-ting→Yuan-ching→Jian-ming→Jian-wei	Yu-chi→Yuan-ching→Jian-wei, Jian-ming→Yian-ting

The order of discussing incorrect solutions switching to close to similar methods from the last order

Teachers were provided the opportunities of discussing when is a good time for teachers inviting students who had incorrect solutions in written cases. The data in

Table 7 indicates that three (T1, T3 and T4) of the five teachers arranged students' incorrect answer (as the method 5) in the last order for discussing in public.

During the case discussion, it reveals that teachers did not have ability to identify where students made mistake and why their answers are wrong. For instance, during the case discussion, T1, T3, and T5 did not distinguish the distinction between Jinan-Ming's method and Jian-wei's method. Jian-wei's misconception was caused from misunderstanding of decimals. In the posttest, excepting T2 other teachers no longer put the method 5 at the final, moreover, the teachers were able to explain the meaning underlying each method of solutions and address the reasons of their arrangement from various perspectives.

CONCLUSIONS AND IMPLICATIONS

It is concluded that teachers' knowledge of students' learning was improved by using written cases. Knowledge of students' learning reported in the study included knowledge of identifying students' multiple methods, categorizing students' various methods, predicting students' anticipated solutions, and ordering students' various solutions for discussing in public. Predicting and ordering students' anticipated solutions mean on the basis of students' prior knowledge, correctness of solutions, and contextual development. Thus, the appropriation of predicting students' anticipated solutions and arranging the order of students' various students are the indicators of teachers' knowledge of students learning. Prior to sequencing students' various solutions, teachers required to identify the distinction among the methods and realize the various meanings of various solutions. Thus, teachers' knowledge of students' learning were improved.

The reasons of the effect of the use of written cases on teachers' knowledge of students' learning could be illustrated as follows. The content of the written cases plays an essential role for improving teachers' knowledge of students' understanding. For instance, students' prior knowledge, students' various solutions coming up in a real teaching, and questions for discussion were covered in each written case. The questions for discussion asked teachers to identify, categorize, and order students' various solutions. Through the case discussion, each teacher was asked to examine carefully the meanings underlying the methods. As a result, teacher recognized that the methods to be predicted were not on the more, rather, on the appropriation. The prediction should rely on students' prior knowledge and cognitive development.

It seems that the teacher with mathematics background has more predication of students' various solutions. For instance, T5 predicted more solutions than T1 and T4. Thus, mathematics background could be a factor of influencing the effect of the use of written cases on improving teachers' knowledge of students' learning. How it affects needs to have a further study in the future.

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