ZDM Mathematics Education (2009) 41:363–378
DOI 10.1007/s11858-009-0175-7

ORIGINAL ARTICLE

Searching for good mathematics instruction at primary school level valued in Taiwan

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Accepted: 12 March 2009 / Published online: 16 April 2009

Abstract In this article, we aim to provide a glimpse of what is counted as good mathematics instruction from Taiwanese perspectives and of various approaches developed and used for achieving high-quality mathematics instruction. The characteristics of good mathematics instruction from Taiwanese perspectives were first collected and discussed from three types of information sources. Although the number of characteristics of good mathematics instruction may vary from one source to another, they can be generally organized in three phases including lesson design before instruction, classroom instruction during the lesson and activities after lesson. In addition to the general overview of mathematics classroom instruction valued in Taiwan, we also analyzed 92 lessons from six experienced teachers whose instructional practices were generally valued in local schools and counties. We identified and discussed the characteristics of their instructional practices in three themes: features of problems and their uses in classroom instruction, aspects of problem–solution discussion and reporting, and the discussion of solution methods. To identify and promote high-quality mathematics instruction, various approaches have been developed and used in Taiwan including the development and use of new textbooks and teachers' guides, teaching contests, master teacher training program, and teacher professional development programs.

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Keywords Good mathematics instruction · Master teachers · Primary school · Taiwan · Teaching contest

1 Introduction

It has been well documented in several large-scale international mathematics studies that East Asian students have superior performance in school mathematics (e.g., Kelley, Mullis, & Martin, 2000; Mullis, Martin, Gonzalez, & Chrostowski, 2004; OECD, 2007). Efforts to search for possible contributing factors have led to the contention that students' high achievement is attributed in part by the cultural context in which students learn. Various factors have been identified in a cultural context, which include mathematics curriculum, parental commitment to their children's education, teacher preparation and in-service teacher support, and the importance of mathematics for every student's successful future (e.g., Kelley, Mullis, & Martin, 2000; Martin, Mullis, & Chrostowski, 2004). Inevitably, recent large-scale international studies have also focused on mathematics classroom instruction in which student learning is involved (e.g., Stigler, Gallimore, & Hiebert, 2000). With Japan as the only education system selected from East Asia in the TIMSS 1995 video study (Stigler & Hiebert, 1999), the researchers found dramatic differences in mathematics classroom instruction sampled from Germany, Japan, and the United States. The results led the researchers to conclude, albeit limited to one high-achieving education system selected from East Asia, that teaching is fundamentally a cultural activity (Stigler & Hiebert, 1999).

The findings from TIMSS 1995 video study suggest the importance of examining and understanding mathematics classroom instruction as situated in different system and social–cultural contexts. Mathematics classroom instruction in Taiwan is not the same as that in Japan (e.g., Stigler et al., 2000). Although some studies documented mathematics instruction practices favorably in general for the case of Taiwan (Lin, 2002), much less is known about the quality of mathematics classroom instruction valued in Taiwan. Given the fact that Taiwan also has a high-achieving education system located in East Asia, much remains to be understood about the nature of mathematics instruction excellence that contributes to students' high achievement in that system. In this article, we thus aim to search for possible patterns of good mathematics instruction that is valued in Taiwan. In particular, two research questions are to be addressed: (1) what are possible characteristics of good mathematics instruction from Taiwanese perspectives? (2) What approaches and cultural resources are employed in Taiwan for helping teachers in their pursuit of effective mathematics instruction?

The following sections are organized into four parts. In the first part (Sect. 2), we provide general background information about mathematics classroom instruction and its development in Taiwan, together with a brief introduction of traditional instruction approach and contrast it with recommended instruction in the context of current curriculum reform. By providing such background information, we intend to outline the common characteristics of good mathematics instruction from Taiwanese perspectives in Sect. 3. A case study of mathematics classroom instruction is carried out in detail to illustrate features of good mathematics classroom instruction. Section 4 is then followed with discussions of various approaches developed and used in Taiwan for pursuing good mathematics instruction. In Sect. 5, we discuss the motivating factors behind the pursuit of good mathematics instruction valued in Taiwan.

2 General characteristics of mathematics classroom instruction and its development in Taiwan

Aspects of the typical classroom instruction in some education systems, such as the US, Japan and Germany, have been extensively studied through video-taped classroom instruction analysis. However, those of Taiwanese classrooms in international contexts have not been studied extensively (e.g., Clarke, Keitel, & Shimizu, 2006; Stigler & Hiebert, 1999). This section will first introduce the typical mathematics instruction recommended in traditional curriculum issued back in the 1970s (Ministry of Education, 1975) and then the learner-centered approach recommended in the innovative curriculum issued around the turn of this century (Ministry of Education, 1998).

2.1 Traditional mathematics instruction in Taiwan

Diverse aspects of valued teaching are articulated and recommended in mathematics curriculum documents in many education systems (e.g., NCTM, 1991, Ministry of Education 1993). There is no exception for Taiwan, where traditional classroom instruction as emphasized in the mathematics curriculum of 1970s was featured as offering well-organized teacher-directed instructions and accompanied with contrived paper-and-pencil tests.

In particular, most teachers teach by following unified textbooks and teacher's instructional guides lesson by lesson. The major role that teachers play is to help students pass a quiz. Teachers hardly take individual students' educational needs into account. A typical pattern of traditional mathematics instruction is often characterized as teacher-centered, content-oriented, examination-driven, whole-class teaching, with no interactions between students and the teacher. As a result, memorization and drilled practices are highly emphasized, while meaningful understanding of mathematical concepts, problem solving, reasoning, and mathematical connections tend to be overlooked. Examination-driven culture also contributes to school teachers' preference for the teacher-centered approach, because the approach shows certain advantages in classroom management when teaching in a large-size class. Because the teacher-centered approach allows the teacher to cover more content in classroom instruction, more teachers have a preference for the teacher-centered approach as the grade level goes up. Therefore, there are more high-school classrooms taught with the teacher-centered approach than primary school classrooms.

2.2 Recommended mathematics instruction

With the new mathematics curriculum standards enacted a few years ago, innovative classroom teaching has been recommended to make each classroom a mathematical learning community. Unlike traditional teacher-centered approach, a learner-centered approach has been recommended in the new mathematics curriculum issued by the Ministry of Education (MOE) (Ministry of Education, 1993; 1998). The learner-centered approach expects to engage students with cooperative learning rather than simply to have a collection of individuals in the classroom. The correctness of students' answers should be verified by logic and mathematical evidence rather than determined by teachers as the sole authority. Effective teaching in the recommended approach requires that teachers know how to ask critical questions and plan lessons that connect with students' prior knowledge, create mathematical tasks and analyze students' learning in order to make ongoing instructional decisions, and stimulate classroom discourse so that students are clear about what is being learnt. Teachers are expected to move toward questioning and listening and away from telling students what to do. The role of teachers is shifted from being a problem solver to being a problem poser.

However, the above-stated expectations do not provide a prescription of what counts as good mathematics instruction. In fact, specific characteristics of good mathematics instruction likely vary from one teacher to another and the nature of content topics being taught (e.g., Li, 2004). Teacher educators and researchers in Taiwan have put more attention on supporting teachers to develop the learner-centered classroom instruction recommended in the new mathematics curriculum than identifying specific characteristics of good mathematics instruction. Mathematics educators are devoted to seeking good strategies for helping teachers in developing their own high-quality mathematics instructions that is articulated in the new curriculum (Lin, 2002). Nevertheless, although mathematics educators and researchers do not take the risk to specify the characteristics of a good mathematics instruction, they implicitly have their own criteria of what can be counted as good mathematics instruction when they are developing various approaches for improving the quality of mathematics instruction. In the next section, we will explore possible characteristics of good mathematics instruction that is valued in Taiwan.

3 Characteristics of good mathematics instruction valued in Taiwan

To examine and summarize what may be counted as high-quality classroom instruction in Taiwan, we reviewed literatures and relevant documents together with a case study. In particular, we searched the Internet and published papers to find literatures on effective mathematics instruction from a Taiwanese perspective. The data collected in this stage included: some published papers, the document of Teacher Professional Development Evaluation issued by the Ministry of Education (Ministry of Education, 2008); and the criteria commonly used in classroom observation to evaluate the quality of a good mathematics lesson.

To illustrate specific characteristics of good mathematics instruction valued in Taiwan, we also analyzed classroom instruction by a group of six experienced teachers who have commonly been recognized as good teachers. These teachers were selected from a teacher professional development program that is accredited by outsiders as helping to develop high-quality mathematics instruction. In particular, this professional program has been ranked continuously as one of the top 5% of the research proposals submitted to the National Science Council (NSC) in Taiwan over the past 10 years. Within 10 years, most of the participating teachers in the program have been authorized as master teachers in various counties. Several teachers have also been awarded the prize of creative mathematics instruction. In particular, one of the teachers in the group (Ms. Jing)[1] was also chosen to illustrate the type of classroom practice in discussion, if needed. Ms. Jing has been participating in the program for 9 years. Her successive participation is partially due to her commitment to the learner-centered instructional approach underpinned in the program.

3.1 General characteristics of good mathematics instruction that are commonly perceived

As mentioned above, what counts as good mathematics instruction is not readily available in detailed description. In fact, no literature seems to be produced in Taiwan apparently articulating specific features of good or excellent mathematics instruction. Nevertheless, the aspects of good mathematics instruction to be achieved can be found in several existing studies and official documents. In this sub-section, the characteristics of good mathematics instruction are reviewed and summarized with information from three sources: (1) published articles, (2) official documents, (3) school teaching demonstration.

3.1.1 Characteristics highlighted and discussed in published articles

Using "good mathematics instruction" as a key word, a search of library Index and Dissertation Abstracts written in Chinese turned out that there is no such literature available in the database. In the follow-up search with the key word of "professional standards of mathematics teachers (in Chinese)", only two papers were found (Lin & Tsai, 2007; Liu, 2007). When the key word of "effective instruction (in Chinese)" was used, only one paper emerged from the database. That is, effective instructional behaviors across subject contents were sorted by before, ongoing, and after teaching (Lin 2000). Lin structured features of effective instruction in three phases of instruction: planning, teaching, and assessment.

The aspects of a good teaching developed in various studies on professional standards for mathematics teachers are not really the same. For instance, the aspects consist of curriculum, teaching and learning, and assessment (Liu, 2007). Liu suggests that good mathematics instruction requires teachers to critically analyze and restructure teaching materials to build upon students' prior knowledge

[1] All the names used in this article are pseudonyms.

and experiences. The teachers need to master the scope and sequence of the contents to be taught, conduct timely assessment, and give students feedback based on the assessment results. Compared to Liu's study, Lin and Tsai (2007) attend more to students' learning. There are also more items listed in each aspect in Lin and Tsai's study than those in Liu's study.

Generally speaking, the aspects that are recommended for having a good classroom instruction across subjects include understanding students' prior knowledge and experience, setting the scope and sequence of the contents to be taught, creating a classroom atmosphere that promotes teacher–students interactions, asking critical and possibly follow-up questions to engage students and clarify their thinking, using timely assessment, providing students effective feedback with the assessment result.

In addition, some other characteristics mentioned in Lin and Tsai's (2007) and Liu's (2007) studies include: understanding the framework of school mathematics curriculum, posing contextual problems, providing students the opportunities of solving problems and communicating their solutions, and guiding students in mathematical explorations.

3.1.2 General characteristics recommended in official documents

"Grades 1–12 Teacher Professional Development Evaluation" (TPDE) is a recently published official document. The Ministry of Education is piloting the ongoing system of TPDE, but it has not been formally carried out yet (Tsieng, Chang, Chang, & Shiu, 2006). The evaluation system expects teachers to achieve the anticipated 22 criteria including six in lesson design, eight in teaching strategies, three in classroom management, and five in assessment, respectively, as depicted in Table 1.

3.1.3 General characteristics specified and used in school teaching demonstration

Developing and achieving high-quality teaching is a major goal of school teachers in Taiwan. To achieve this goal, experienced teachers in each school are often invited in turns by the school to demonstrate good model of classroom instruction for their colleagues each semester. The criteria used in specifying a good teaching across schools are quite similar but also bear some variations in their

Table 1 Aspects of good teaching recommended in official documents and used school teaching demonstration

Aspects	Performance of lesson	TPDE	STD
Lesson design	Use students' prior knowledge and skills to plan instruction	○	–
	Target on students' misconceptions in lesson planning	○	–
	Understand the scope and sequence of the contents to be taught	○	○
	Plan lesson with creativity	–	○
	Design learning activities to provoke students' thinking	○	–
	Plan to use assessment in examining students' performance	○	–
	Use lesson objectives to plan instruction	○	○
Teaching strategies	Use various techniques to motivate students' learning	○	○
	Present content to students with a well-organized sequence	○	–
	Clarify students' misconceptions	○	–
	Ask critical question for students to reflect on their thinking	○	○
	Use multiple examples in teaching	○	○
	Have good transition from one activity to another	○	–
	Utilize internet, information communication technology (ICT) and manipulatives	○	–
	Integrate the resources of communities into instruction	○	–
Classroom management	Create a learning environment	○	○
	Manage students' interruption	○	○
	Use various techniques in classroom management	○	○
Assessment	Provide useful and timely assessment	○	–
	Use alternative assessments based on instruction needs	○	–
	Analyze students' data to explore how to assess more effectively	○	–
	Use classroom assessment to assist decision making about what and how to teach	○	–
	Use assessment to give effective feedback to students and parents	○	–

Small circle means the inclusion of that aspect in TPDE or STD

TPDE Grade 1–12 Teacher Professional Development Evaluation, *STD* school teaching demonstration details. As an example, the criteria set up and used in Shi-Men Primary School in Taiwan are displayed in Table 1. Based on the criteria, each observer is expected to give the teacher feedback right after his or her teaching. Different from studies discussed in Sect. 3.1.1 and TPDE, school teaching demonstrations do not pay specific attention to assessment. Instead, they attend to the teacher's classroom management and students' responses in classroom instruction.

Taking criteria used in school teaching demonstration as teachers' perspective, the characteristics expected for having a good classroom instruction across subjects consist of understanding students' prior knowledge, setting up clear and adequate lesson objectives, having adequate scope and sequence of the contents to be taught. These features are specified for the phase of lesson planning. The teaching strategies displayed in good classroom instruction include that the teacher is able to use various techniques or skills to motivate students to learn, ask critical key questions to provoke students' thinking, and use multiple examples in explanation.

Taken together, characteristics identified as important parts of good mathematics instruction in Taiwan can be summarized as the following features in three phases. At the phase of lesson planning, the teacher is expected to understand his/her students' prior knowledge and possible misconceptions, set up clear and adequate lesson objectives, and have adequate scope and sequence of the contents to be taught. During the teaching phase, the teacher is expected to be able to pose contextual problems, use various techniques or skills to motivate students to learn, ask critical and follow-up questions to provoke students' thinking, use multiple examples in explanation, provide students the opportunities of solving problems and communicating their solutions, guide students in mathematics explorations, and create a classroom atmosphere that promotes the teacher–students interactions. After teaching, the teacher is expected to provide useful and timely assessment and use the assessment result to give students feedback as well.

3.2 Searching for good mathematics instruction: a case study

3.2.1 Participants and context of the case

Having a list of characteristics may not be enough to show the nature of good mathematics instruction valued in Taiwan. In order to illustrate possible pattern of good mathematics instruction in Taiwan, we used a case study approach.

The classroom instruction of six teachers who participated in a teacher professional development program were observed and analyzed. The professional development program provided teachers unofficial courses to enhance their knowledge of students' learning and then to improve their classroom instruction practices. The six teachers supported themselves mutually. Their classrooms were scheduled for observation in turns. A routine weekly group meeting was scheduled immediately following a classroom observation, and the group meeting was used to discuss the classroom observation throughout the entire year. One content unit's instruction (about 5–8 lessons) for each teacher per semester was videotaped and transcribed. A total of 92 videotaped lessons were used as the primary data in this case study. The 92 lessons include 16, 16, 15, 15, 15, and 15 lessons collected from six teachers' [i.e., T1 (Ms. Jing), T2, T3, T4, T5, and T6] classrooms, respectively. Each teacher was also individually interviewed three times per academic year to trace their instruction. Additional data included the transcription of weekly group meetings in which the participating teachers met routinely to discuss the issues related to mathematics, students' learning, and pedagogy.

All six teachers in this program desired to structure their classroom instructions along the recommended learner-centered instructional approach. In fact, their typical lessons shared a similar instructional flow as follows: (1) reviewing previous lesson, (2) posing the problems for the day, often the problems are built upon the previous day's work, (3) solving the given problem individually or in groups, (4) inviting students to explain their solutions and thoughts, (5) summarizing the significant idea(s) of the lesson.

In addition to examine the general structure of good mathematics instruction embedded in these six teachers' practices, we plan to illustrate the meaning behind the general structure of good mathematics instruction with the case of one participating teacher's classroom instruction. Ms. Jing, selected as such a teacher for possible illustration, has the longest years' involvement in the program. She has been identified as the teacher with the best quality of mathematics instruction in the program by the program organizer and other participating teachers. Ms. Jing has been teaching in primary school for 19 years and has a master's degree in mathematics education. She has also been invited to teach the course of Mathematics Method of Teaching in the University of Education for several years. She won the award of "power teacher" in 2003 and the prize of "excellent" teacher in 2008. The award of "power teacher" is highly reputed in school mathematics instruction. She has been a master teacher for 6 years in a city. In Taiwan, the master teacher is a consultant of mathematics instruction for other school teachers. Besides, she is the consultant when her colleagues encountered difficulties and issues related to mathematics classroom instruction.

3.2.2 Method of case analyses

To search for possible patterns of good mathematics instruction embedded in these six participating teachers' practices, the six teachers' videotaped lessons and interview data were analyzed and coded. All the data analysis was carried out in the original language of Chinese. Selected data were translated to English to provide evidence in the later sections of this article.

Our analysis of teachers' pre-instruction practices relied on on-site observation, group discussion recording and interview data. A holistic analysis was conducted to identify possible approaches and emphases in these teachers' practices for developing effective classroom instruction.

For teachers' videotaped lessons, the data analysis was a process that integrates iterative lesson instruction examination and code development. Because these teachers shared a similar instructional routine with a learner-centered approach, a consensus was reached to focus on these teachers' practices in involving and guiding students in problem-solving activities in their lessons. A coding schema was then developed with 27 categories in three themes. The three themes include features of problems and their uses in classroom instruction (16 categories), aspects of problem–solution discussion and reporting (4 categories), and the discussion of solution methods (7 categories). For instance, 16 categories for the theme of problem features and uses include: sources of problems (5), problem context (3), ways of presenting problems (4), and anticipated uses of problems (4). The data were coded by one researcher and four teachers with a master's degree in mathematics education. Each code was counted and frequencies were recorded. The inter-rater agreement for coding each aspect was 85% and above. All discrepancies were resolved through discussions.

3.2.3 Patterns of the six teachers' pre-instruction practices for developing a good mathematics instruction

Prior to the pre-scheduled teaching, all six teachers in the professional development program had similar instructional preparation to develop a better understanding of their students. Before teaching, they conducted pretest for students and reviewed literatures in order to know better about students' learning of a specific mathematics topic that is to be taught. In addition, exploring and analyzing various series of textbooks for comparing and contrasting the contents, pedagogy, and sequences of activities are routine work among these teachers in their lesson preparation.

Textbook analysis plays an important role in their lesson planning. The teachers are used to conjecturing students' learning trajectory together with critically analyzing textbook content arrangements. The planned teaching trajectory of a lesson corresponding to the hypothetical students' learning trajectory is actually conjecturing teaching trajectory, because the teachers learned from previous lessons with successive revision stemmed from the change of students' learning trajectory. Thus, conjecturing teaching trajectory is provisional, tested, and modified from one lesson to another. The conjecturing teaching trajectory is not finalized until just 1 min before the lesson to be taught. It is called "conjecturing" because teaching trajectory is not a trial and error work. Rather, it is based on students' performance in previous lessons.

3.2.4 Characteristics of good mathematics instruction featured in these six teachers' practices

During classroom teaching, a common instruction flow was presented in these teachers' typical lessons. It started with reviewing previous lesson, giving student problems to solve, students working individually or in groups, and moving forward to discuss students' various solution methods, and finally highlighting the main point.

Reviewing previous lessons to refresh students' prior knowledge with respect to the content to be learned is a common instruction phase at the very beginning of a lesson. Correcting students' misconceptions presented in the previous day's assignment is often done during the phase of reviewing previous lesson.

The characteristics of good mathematics instruction of the six teachers' classrooms can be highlighted in three themes: features of problems and their uses in classroom instruction, aspects of problem–solution discussion and reporting, and the discussion of solution methods.

3.2.4.1 Features of problems and their uses in classroom instruction The problem selection and use are commonly perceived as important aspects for developing effective classroom instruction. In our analysis of these six teachers' problem selection and uses, we identified four aspects in our analyses. They include the sources of the problems used, problem context, ways used to present problems, and the possibility of invoking students' specific solution methods. The frequencies coded from the six teachers' 92 lessons in terms of problem features and uses are depicted in Table 2.

The results show that a total of 276 problems were used by the six teachers in these 92 lessons. On average, three problems were provided and used by a teacher in each lesson. In the following sub-sections, further results are provided in terms of four characteristics identified through data analysis.

Characteristic 1: teachers' efforts and the textbook were the main sources for problems used in classroom instruction.

Table 2 Features of problems and their uses in classroom instruction

Problems' features and uses	T1	T2	T3	T4	T5	T6	Total
Sources of problems							
Selected from a textbook	5	6	6	7	8	10	42
Revised from a textbook	27	18	27	27	24	30	153
Designed by the teacher	11	13	9	8	9	5	55
Generated by students	2	5	2	1	3	0	13
Generated by the teacher and students together	3	6	1	2	1	0	13
Total	48	48	45	45	45	45	276
Contexts							
Pure math problems	0	0	0	0	0	0	0
Problems provided verbally	0	0	0	0	1	2	3
Real life problems	48	48	45	45	44	43	273
Ways of presentation							
Verbalized by instructor only	48	48	42	45	42	40	265
PowerPoint or Over-Head Projector	0	0	6	0	3	0	9
Blackboard or whiteboard	48	48	42	45	42	40	265
Read by students only	0	0	6	3	6	8	23
Anticipated use							
Fulfilling instructional objectives	47	46	44	44	43	42	266
Relating to students' prior knowledge	48	48	45	45	45	45	276
Eliciting a specific solution strategy	12	10	9	7	7	4	49
Evoking multiple solutions	42	41	38	37	35	30	223

The textbook was the main source for these teachers to select and develop mathematics problems. Of the 276 problems presented in 92 lessons, 153 problems were revised from these teachers' textbook. The teachers realized the fact that having too many problems may not lead to a better result of facilitating students' learning with understanding. Thus, they gave only one or two problems at the first two lessons of a content unit, because the first two lessons are usually designed to help students learn fundamental concepts. As usual, the number of the problems used in the latter lessons was increased to an average of three or more problems per lesson for teaching and learning procedural skills.

The teachers tended to pay much attention to the numerals in the problem, the problem's semantic structure, the correspondence between the lesson's instructional objective and its activity, and the sequencing of these problems when revising the problems from the textbook. For instance, Ms. Jing (T1) attended to the numerals presented in a problem related to fractions. She wanted to ensure that the numerals used in a problem are not too large for operating two fractions. Smaller numbers would make students partition a fraction into equal parts with ease.

Sequencing problems on the basis of students' cognition played an important role for these teachers. For instance, Ms. Jing sequenced the problems of finding a fraction by expanding a denominator prior to those by reducing a denominator. Students' learning of equally partitioning small parts as corresponding to expanding denominator is easier than regrouping the parts as corresponding to reducing denominator. For example, problem (a) was presented prior to problem (b) as follows.

> Problem (a): a box has 40 apples. The amount of apples of $\frac{1}{4}$ of the box is as many as those of $\frac{()}{8}$ of the box. What is the number in ( )?

> Problem (b): a box has 40 apples. The amount of apples of $\frac{8}{20}$ of the box is as many as those of $\frac{()}{5}$of the box. What is the number in ( )?

Characteristic 2: the teachers have a strong preference of selecting and using problems with a real-life context.

It is clear that the teachers had a preference of selecting and using problems with a real-life context. In fact, 99% (273 out of 276) of the problems were contextualized. They believed that contextualized problems are more meaningful for the students than pure mathematics problems. Therefore, contextual problems are more likely to contribute to students' meaningful learning. For instance, Ms. Jing (T1) did not give students $\frac{3}{4} = \frac{()}{100}$ (pure mathematics problem) to solve. Instead, she used a contextualized problem:

The auditorium has 300 people, $\frac{3}{4}$ of the people in the auditorium are as many as $\frac{()}{100}$ of the people in the auditorium. What number is in the ( )?"

Characteristic 3: the problems were most frequently presented on the blackboard together with the teacher's verbal explanation.

The total of 276 problems were presented in several different ways: reading a problem on the textbook by students, writing it on the black or white board by the teacher together with his/her verbal explanation, and presenting a problem on the OHP or PPT accompanied with students' or the teacher's explanation. 96% (265 out of 276) of the problems were presented on the black or white board and read by the teacher. This is the most common method used by these teachers to present problems, as they believed that such presentation readily contributes to students' comprehension. These teachers also realized that the problems presented with a teacher's verbal explanation but without writing on the blackboard would not help students catch up the information completely for solving the problems in a short time.

Characteristic 4: the problems were selected and used with a clear intention as of relating to students' prior knowledge, fulfilling a lesson's instructional objectives, and evoking multiple solutions.

As shown in Table 2, four aspects were considered by these teachers when they selected and used the problems. The results indicate that the problems selected and used by teachers tended to relate highly to students' prior knowledge (276 out of 276 problems) and their lesson's instructional objectives (266 out of 276 problems). Many problems were also used with the expectation for students to develop multiple solutions (223 out of 276 problems). However, the design and use of the problems for eliciting anticipated solutions (49 out of 276 problems) seemed to be a challenge for these teachers.

For instance, the instructional objective of "finding equivalent fraction by reducing numerals" can be easily achieved by the majority of teachers via introducing the algorithm, such as "A paper strip is 8 meters long, what fraction is $\frac{4}{8}$ of the strip reduced into?" Conversely, Ms. Jing argued that the problem was inappropriate because students did not yet learn the terminology of "reduced fraction". Therefore, she revised the problem and restated it as "A paper strip is 8 meters long, what proportion of the strip has the same length as $\frac{4}{8}$ of the paper strip?" Moreover, in order to elicit the reduced numerals strategy, the problem that Ms. Jing proposed is "A paper strip is 8 meters long, what proportion of the strip has the same length as $\frac{1}{2}$ of the strip?" Ms. Jing was aware that the size of denominators and 8 m in the problems need to help elicit anticipated students' solutions. To help evoke multiple solution methods with the use of both expanded and reduced numerals strategies, the problem was further revised as "A paper strip is 8 meters long, what proportion of the strip has the same length as $\frac{2}{4}$ of the strip?

3.2.4.2 Aspects of problem–solution discussion and reporting in classroom instruction Once a problem was presented, students were frequently asked to solve it individually. Sometimes, students worked together in groups if needed. Once group work was needed, students worked together more in heterogeneous groups than in homogenous groups. The teacher routinely circulated the classroom to see students' problem-solving progress. The teacher was also looking for all possible solution methods emerged and differentiated them during students' work either individually or in groups.

Identifying and selecting students' various solutions were an essential work before classroom discourse started. There were two characteristics focused in our analyses in terms of the focus and approach of whole-class discussion of students' solutions. The two characteristics as shown in Table 3 consist of identifying and selecting solutions for discussion (two categories), and solution reporting (2).

Characteristic 1: identifying and selecting students' various solutions for the whole-class discussion with a focus on both the problem solution and its process.

These teachers selected students' various solutions for discussion by considering if the solution is wrong or right, the solution process, and the solution utilized by the number of students. Table 3 shows that these teachers cared about both the problem solution itself and the solution process, when selecting them for the whole-class discussion.

In general, these teachers tended to first ask the students who gave incomplete or wrong answers to explain their solution and methods, and then followed by those who had right answers (for 205 out of 276 problems). Sometimes, if

Table 3 Focus of problem–solution discussion and its reporting in classroom instruction

Aspects	T1	T2	T3	T4	T5	T6	Total
Selection of solution							
In terms of solution correctness	48	48	45	45	45	45	276
In terms of solution process	48	48	45	45	45	42	273
Solution reporting							
Instructor	0	0	0	0	0	0	0
Students	48	48	45	45	45	45	276

the solutions are too complicated to be articulated clearly, these teachers preferred to invite students with a correct solution to report first, and then followed by those who had wrong answers (for 71 out of 276 problems).

In fact, these teachers also looked up into the quality of the solutions, such as the use of multiple representations and students' conceptual development. The teachers tended to pay close attention to solutions with multiple representations, then to sequence the class discussion of solutions from the use of simple representation to complex one. Students' hierarchy of conceptual development was also an essential factor when these teachers decided the order of solutions to be reported, such as from students' prior concept to the more complicated new concepts to be learned in current lesson. Certainly, the aspect of students' conceptual development was sometimes overlooked in some classrooms.

Characteristic 2: the solutions were always reported and shared by students not the teacher.

There was an obvious pattern that the solutions were reported by the students after the teacher made the selection of solutions. None of these teachers took the responsibility of reporting selected students' solutions (see Table 3). Instead, the teachers always asked the students to present and share their solutions with the whole class.

For instance, in a typical lesson, Ms. Jing (T1) encouraged students to solve problems in any way they wanted. After students came up various solutions for a problem, she briefly reflected on each of 32 students' solutions and sorted them into different categories. She then quickly selected one from each category and organized them in a sequence for discussion. She tended to invite students who did not get a correct answer to present their work to the whole class. The solutions with wrong, incomplete, to right answers were successively reported for the whole-class discussion. The number of solutions to be selected to report in public also depended on the categories of distinction among various methods.

Finally, these teachers invited students to explain their solutions one by one in order to attract and maintain students' attentions. In order to help students identify possible connections and differences among various solutions, these teachers also led students to review various solutions after each solution was reported.

3.2.4.3 Discussion of problem solution methods The teachers noticed that in order to develop students' speaking and thinking mathematically, working individually or in groups were not enough. Whole-class discussion was used as an important phase in these teachers' classroom instruction. After each problem was solved, students were constantly invited to talk and think mathematically at the phase of whole-class discussion. The results show that after sequencing various solution methods, the teachers were skillful in organizing and orchestrating classroom discourse. Seventeen categories in the theme of discussing solution methods consists of questioning (7), teachers' attitude toward students' questions (2), dealing with students' misconceptions (5), and the interaction of instructor-students (3).

There were three characteristics identified from the discussion of solution methods. They include teachers' frequent questioning, very limited number of students' misconceptions and their self corrections, and teachers' skillful discourse with students. Summarizing and highlighting important mathematical ideas are presented as the fourth characteristic that is often presented at the end of whole class discussion.

Characteristic 1: the teachers often asked students various questions during the process of discussing students' solutions and methods used.

After students explained their methods of solutions, the teachers' follow-up talks were devoted to ask students many questions with various purposes. The teachers' questioning in the discourse was to ask for clarification of students' mathematics thinking and to encourage students to solve problems with multiple methods. In particular, teachers questioned students to clarify how they get their answers, encourage students to explain their reasoning, ask students to distinguish one solution or thought from another, diagnose students' misconceptions, and help to correct students' misconceptions.

For instance, after encouraging students to produce, compare, and contrast multiple solution methods, Ms. Jing moved her focus to help students identify certain methods with salient advantage of achieving specific objectives over others. In particular, to develop students' awareness of the relationship between numerator and denominator of equivalent fractions, Ms. Jing argued that the relationship must be built upon a line-segment model rather than the circle model. Students were given the problem (c) as follows. As we noticed, many students solved the problem with several solution methods using non-line segment models. After selecting students' various solutions, Ms. Jing invited students to explain their thinking behind the methods of solutions. She then selected one solution method with the use of a line segment from various methods and let students observe the change of partitioning numerator and denominator.

Problem (c): a box has 40 apples. The amount of apples of $\frac{1}{4}$ of the box is as many as those of $\frac{(\ )}{20}$ of the box. What is the number in ( )?

Table 4 Types and frequencies of teachers' questioning in discussing solutions

Questioning	T1	T2	T3	T4	T5	T6	Total
Ask for solution explanation	235	226	212	208	207	205	1,293
Clarify students' ideas	35	19	32	38	33	30	187
Ask for solution completion	60	55	55	54	48	45	317
Compare solution similarities and differences	113	106	99	100	96	100	614
Diagnose students' misconceptions	20	16	13	14	5	3	71
Ask to summarize main concepts	48	48	45	43	40	42	266
Ask to clarify students' misconceptions	24	19	16	12	8	9	88
Total	535	489	472	469	437	434	2,836

Table 4 shows that the questions asked for students' explanation of their thinking were the most common type of questions. There were 1,293 questions asked by these teachers that invited students to explain their solutions in 92 lessons.

Table 4 also shows that the number of questions asked by T1 (Ms. Jing), T2, T3, T4, T5, and T6 at the phase of discussing solutions were 535, 489, 472, 469, 437, and 434, respectively. In other words, the number of questions asked by T1 (Ms. Jing), T2, T3, T4, T5, and T6 after solving each given problem are on average 11, 10, 10, 9, 9, 9, respectively. The average of 9–11 questions was asked to diagnose and clarify students' misconceptions, and help students identify possible similarities and differences in their thinking and solutions. The questions for making up students' incomplete solutions or thinking were asked by the teachers as well. The questions for diagnosing students' misconceptions seemed to be the most challenge for these teachers.

Characteristic 2: occasional misconceptions were noticed by teachers, but corrected by students rather than teachers.

Except for obvious errors or incomplete solutions, there were a total of six students' misconceptions occurred in the 92 lessons. While five out of six misconceptions were pointed by teachers, one was realized by the reporting student himself. However, all these misconceptions were corrected by either students or the reporting student. None of these misconceptions were corrected by teachers.

Characteristic 3: the teachers always attended to the reporting students and sometimes other students, but would not dominate the discourse with students.

The teachers played an important role in developing and nurturing classroom discourse. However, they would not dominate the discussion with students. Moreover, they were often successful in orchestrating in-depth discussion with students. In addition to asking students to explain or illustrate their own solutions, these teachers also frequently asked other students to comment on the solutions. This is one way used by these teachers to create more opportunities to engage students into discussions and interact with more students. As illustrated in the following episode, Ms. Jing presented the following problem (d). Her students solved the problem using multiple solution methods. Then, Ms. Jing asked students to compare and identify which method is more accurate and efficient.

Problem (d): a box has 24 pieces of chocolates. Sue has $\frac{1}{4}$ box. Steve has $\frac{3}{12}$ box. Who has more between Sue and Steve? How do you know? Explain.

In Fig. 1, possible solutions using three different approaches are presented, as done by three students (i.e., David, Susan, and Tom).

David	This is Sue's. $\frac{1}{4}$ of a box has 6 pieces of chocolates. So Sue has 6 pieces. Steve has $\frac{3}{12}$ of a box. I partitioned the whole box into 12 parts. Three parts has 6 pieces of chocolates. So that Sue and Steve have the same amount of chocolates
T	There are three methods here. Is each of them reasonable? David comes up first to speak up loudly
T	Let us see the next one
Susan	I partitioned the whole into four parts. This represents Sue's
T	What stands for a box in your drawing?
Susan	The whole circle represents a box. The other circle standing for Steve's was partitioned into 12 equal parts. I shaded three parts as $\frac{3}{12}$. The two shaded areas are same. Thus $\frac{1}{4} = \frac{3}{12}$
T	Can any of you identify if the two shaded areas are equal?
All	No
T	It is hard to identify their shaded areas are congruent right? Let us see Tom's solution. Is it clear enough?
All	Yes

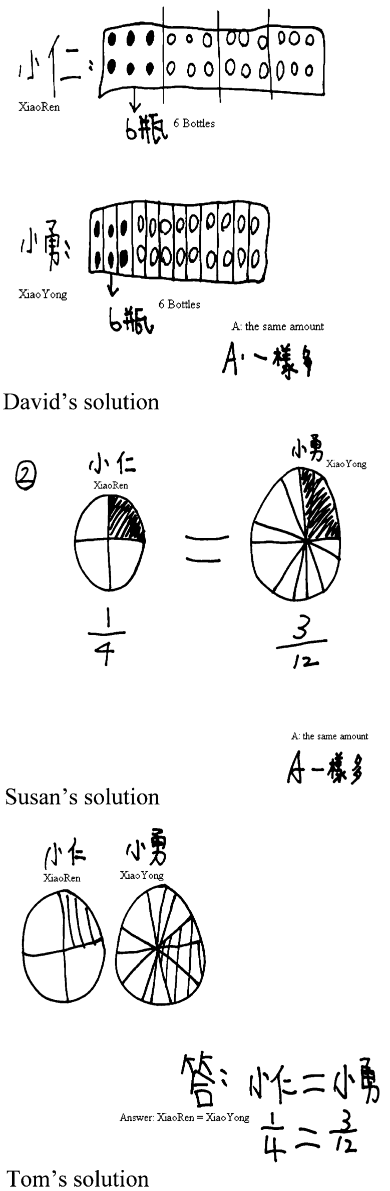


Fig. 1 Three different solution approaches used by David, Susan, and Tom

Tom	This is the area of $\frac{1}{4}$. That is the area of $\frac{3}{12}$. Their areas are congruent
T	Do you think the two shaded areas are equal?
All	Yes

In general, the reporting student only interacted with the teacher in normal classrooms. However, the reporting students in classrooms of these teachers who participated in the teacher professional development program frequently interacted with the teacher as well as some other students.

Characteristic 4: highlighting and summarizing main points at the end of the discussion.

Approaching to the end of a lesson, these teachers often summarize important mathematical ideas and main points. For instance, T1 (Ms. Jing) highlighted the importance of understanding the meaning of equivalence of two fractions instead of an emphasis on algorithm. She emphasized repeatedly the relationship between numerator and denominator of two equal fractions.

In these teachers' classes, the comparison and contrast of multiple solution methods to a given problem were also briefly made at the end of a lesson. The comparison of various solution methods was not focused on efficiency. Instead, the comparison was to identify which method is more likely and easily to achieve the objectives of a lesson. For instance, to promote students' advanced thinking, Ms. Jing selected a student's solution that has the advantage of seeing the relationship between the numerator and denominator. Hong, one of the students, first partitioned the line segment into four equal parts, each part representing $\frac{1}{4}$ length of the rope, and then each part was re-partitioned into five equal subparts and thus subdivided the rope into 20 parts. Hong's solution helped other students to perceive $\frac{1}{4}=\frac{5}{20}$, because the numerator and denominator were multiplied by 5 simultaneously through one part partitioned into five subparts and 4 parts partitioned into 20 subparts on the rope, as depicted in Fig. 2.

Finally, the transition from one activity to another as corresponding to students' conceptual development is usually forged at the end of the lesson in these teachers' classrooms. These teachers also tended to extend a lesson with students' assignments as a formative assessment. Students' assignment was used as a lesson's follow up for students' learning and a part of preparation for the next lesson for teachers.

3.2.5 Summary

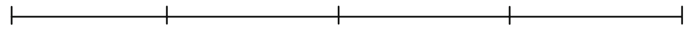
The patterns of good instructional practices identified from the case study analysis above were evolved before, during, and after teaching as follows.

1. Prior to classroom instruction, teachers used to do a detailed preparation in conjecturing teaching trajectory of the lesson based on hypothetical students' learning trajectory. It was a process that includes reviewing literatures, conducting pretest for students, identifying and organizing the lesson's instructional objectives together with critical analyses of the textbook.

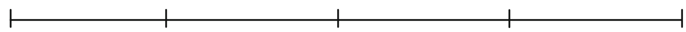
2. In general, the lesson was started by presenting students contextual problems that are relevant to students' real life and prior knowledge. The problems were often presented on the blackboard together with the teacher's verbal explanation. In general, the problems were chosen from the textbook and modified by the teacher to evoke

Fig. 2 Discussion of Hong's solution of $\frac{1}{4} = \frac{5}{20}$ with a line-segment model

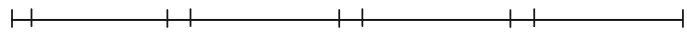
T : This is Hong's solution. She took much time to draw. Tell us how you did it, Hong.
Hong: The whole line stands for a box with 40 pieces of apples. It is partitioned into 4 equal parts.



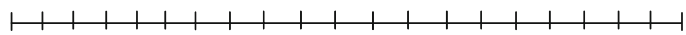
T: Where is $\frac{1}{4}$ of the box?
Hong: (pointed out a part of the line).
T: Do all of you have any question?
All: No.
Hong: [Drawn another line with same length as before]
T: Why did you draw another line?
Hong: I want to figure out $\frac{1}{4} = \frac{(\ )}{20}$.
T: What are going to do next?
Hong: I do the same way as previous one. Partitioning it into 4 parts.



T: What are you going to do next?
Hong: Then, each part is partitioned into one small part.



T: How many small parts are in total now?
Hong: 8 parts
T: Do you know what she is going to do?
S1: Partitioning one part at a time.
T: How many parts are there in total?
All: 20



T: Are you aware of the relationship between the two fractions, $\frac{1}{4}$ and $\frac{5}{20}$, based on the partitioning?

students' multiple solution methods. The mathematical concepts were often introduced through students' mathematical activities instead of the teacher's telling.

During the lesson, students were frequently asked to solve a problem individually and sometimes in groups if needed. Identifying, selecting, and sequencing students' various solution methods were essential for the teacher to do before the whole-class discussion. Selecting students' various solutions for discussion depended on several factors including the solution correctness and its process. Selected solutions for discussion were often sequenced as starting from wrong solutions followed by right solutions, or in terms of various representations used or students' conceptual development.

Students were encouraged to explain and justify what they discovered. The teacher always paid a close attention on the reporting students and sometimes attended to the rest of the students, but did not dominate the discourse. The reporting student frequently interacted with the teacher as well as some other students in the classroom. Multiple solutions were presented and explained one by one. Students were afforded with many opportunities to explore the meaning behind algorithms and to connect visual representations with numerical sentences. Students were given the opportunities of comparing and contrasting various solutions in terms of mathematically significant ideas. Students' problem solving and discourses constituted the focus of classroom instruction.

After students presented and explained their solutions, the teacher frequently asked various questions to further clarify, compare, diagnose, and extend students' mathematical thinking. At the same time, the teacher highly respected students' questions that are raised in the lesson and invited other students to respond them. Occasional misconceptions or errors, as often noticed by the teacher, were corrected by students themselves other than the teacher. Approaching the end of the lesson, the teacher was used to summarize important mathematical ideas and points together with students.

3. After the lesson, teachers would extend the lesson with students' assignments as a formative assessment that is also used as a follow-up and a part of preparation for the next lesson. The activities at various phases form an iterative process that is used in developing good mathematics instruction in Taiwanese classrooms.

4 The development of good mathematics instruction valued in Taiwan

Various approaches have been developed and used in the pursuit of high-quality mathematics instruction in different education systems in East Asia. For instance, lesson study is an important method utilized in Japan to improve the quality of mathematics instruction (Fernandez & Yoshida, 2004). The approach of exemplary lesson development has been used in the Chinese mainland to improve the quality of mathematics instruction (Huang & Bao, 2006). Instructional contests are organized to promote quality mathematics instruction in several education systems in East Asia (e.g., Li & Li, 2009; Lin, 2008; Pang, 2008; Shimizu, 2008). Master teachers also play an important role in improving mathematics instruction in the Chinese mainland (e.g., Li, Huang, Bao, & Fan, 2009). However, much remains unknown to outsiders about possible approaches used in Taiwan.

Pursing excellence in classroom teaching has been one of the main goals for the Ministry of Education. Thus, the Ministry of Education in Taiwan persistently provides teachers long-term supports for pursuing high-quality mathematics instruction system-wide, such as mathematics consultants residing in each county. To promote mathematics classroom instruction excellence, various strategies, techniques, and activities have been developed and carried out through different programs.

First, mathematics textbooks together with students' workbooks and teachers' guides have been developed to embody the learner-centered pedagogy recommended in the curriculum reform. As the data reported in TIMSS 2003, 92% of Taiwanese teachers heavily relied on the textbook as the main instructional resources (Lin & Tsai, 2006). Thus, the use of textbooks is considered as a naturally effective strategy for helping teachers to develop the recommended instruction.

To assist teachers in understanding the philosophy and guidelines underpinned the innovations in textbooks and classroom instruction, mathematics educators and textbook developers make considerable attempts to disseminate information through various channels, including professional conferences, various large-scale institutions and workshops across schools and counties. A typical activity used in workshops or institutions is to demonstrate an exemplar of high-quality mathematics instruction. Thus, the demonstration of good mathematics instruction offers teachers important opportunities to learn how to improve the quality of mathematics instruction.

Second, the Ministry of Education or the local educational bureau in each county often organizes various teaching contests for teachers. Such contests have become a formal driving force for identifying and promoting good mathematics instruction. While there may be some variations in terms of teaching contest organization and emphases across local educational bureaus, teaching contests share the common purpose of promoting teachers' instruction development toward high-quality mathematics instruction. Teaching contests provide a unique opportunity for teachers not only to reflect on their own instructional practices, but also learn various good models of mathematics instruction. The teachers who won the first three prizes in contests have been named as "super teachers" and their lesson plans have also been published.

Third, a master-teacher training program has been funded by the Ministry of Education since 2003. The goal of the program is to train a teacher to become a master teacher who is able to provide consulting service in mathematics instruction to other school teachers. Fifty teachers are recruited each year from schools across all counties to participate in the program. Most of these teachers are recommended by schools or the local educational bureau. These participating teachers receive training through a series of institutes or workshops at both the beginning and the end of school semester. The rationale and ideas behind innovative curriculum materials, recommended mathematical instruction, and assessment are primary contents of the institutes and workshops. During the school year, these teachers are assisted in their classroom instruction practices by a group of teacher educators from universities. Teachers are also required to participate in large-scale meetings either across counties or across schools in a county. Now, there are about 6–10 master teachers in each county as a pool of consultants to help improve other teachers' mathematics instruction. In particular, the teachers who participated in the teacher professional program led by the first author are frequently invited to join the group of master teachers. Master teachers are often invited by schools to give lectures, to write textbooks, and to provide professional assistance for improving mathematics classroom instruction. Master teachers are also required to make their lessons publically available to other teachers periodically. They have the obligation of demonstrating a good model of mathematics instruction for other teachers from a school, a county, or even across the entire system. This approach helps to lead other teachers to move toward good mathematics instruction that is currently valued and promoted in Taiwan.

Fourth, it has been an educational tradition in Taiwan to use Wednesday afternoon as teachers' professional development time. Correspondingly, students have no school on Wednesday afternoon. There are various professional development activities available for teachers hosted by schools or educational bureau in every county. For example, lectures are frequently provided by mathematics educators of the Universities of Education, who are invited to give workshops related to primary mathematics teaching and learning. Master teachers are sometimes invited to deliver a lecture related to teaching practices in mathematics. Teachers may be volunteering or required to choose an activity.

Fifth, the teacher education center in each University of Education continually supports school teachers for their professional needs in mathematics instruction year after year. Traditionally, the University of Education has been a place where school teachers looked for help with the difficulty encountered in mathematics teaching. In general, at the very beginning of a school semester, each teacher education center gives surrounding schools a form to fill out what kind of supports in mathematics instruction schools may need. Once the completed forms are returned, the teacher education center assigns mathematics educators at the university to support school teachers in mathematics instruction. Teacher education centers also provide mathematics educators additional stipends for the extra work. Thus, teacher education centers of the Universities of Education have played a significant role in promoting school teachers' development of high-quality mathematics instruction.

Finally, several teachers' professional development programs, funded by the National Science Council, have provided more opportunities of assisting school teachers to develop recommended mathematics instruction. Teacher educators from the University of Education are devoted to research projects of supporting teachers in improving their classroom instruction practices. In fact, what is reported in this article is one of such teacher professional development programs. This program has been continually funded since 1997 and the majority of the program's participating teachers have been accredited as master teachers in mathematics instruction by outside evaluators.

There are six in-service teachers that are recruited from the same grade level if at all possible to participate in the program each year. The same grade level lends similar mathematical content readily as a base of discussion, when the teachers met together after observing each other's lessons to address issues. All participating teachers' lessons were scheduled to be observed in turn. In particular, these teachers were scheduled to sit altogether in a classroom to observe a lesson and immediately have a follow-up of 3-h meeting. The teacher who taught the lesson was asked to reflect on his/her own teaching and the rest of the participants were invited to articulate what they observed in the lesson. Relevant activities normally included critically analyzing textbooks, conducting pre-test, planning a lesson, observing the lesson, immediate follow-up discussing, post-test or homework as a formative assessment. These activities are used to support participating teachers to improve the quality of mathematics instruction.

5 Conclusion

The characteristics of good mathematics instruction revealed from the above case study in Sect. 3.2 share many similarities, but in further detail, with what is commonly perceived in Taiwan (see Sect. 3.1). The patterns of good mathematics instruction shown in the six teachers' videotaped lessons were evolved before, during, and after teaching. Prior to teaching, teachers were skillful in conjecturing teaching trajectory of the lesson through the process of reviewing literatures, conducting pre-test for students, identifying and understanding the lesson's objectives after critical analysis of their textbook. During classroom instruction, the lesson started by providing students contextual problems that relate to students' daily experience and prior knowledge. The problems were often taken from the textbook and revised to evoke students' multiple solutions. After solving problems either individually or in groups sometimes, identifying, selecting, and sequencing students' various solutions were an essential work for the teacher before the whole-class discussion. Students were encouraged to explain and justify what they discovered. Students were given the opportunities for comparing and contrasting various solutions in terms of mathematically significant ideas. After teaching, teachers extended the lesson with homework assignment as a formative assessment, which was also used as a part of the preparation for the next lesson. The activities at various phases formed an iterative process that helps to develop good mathematics instruction valued in Taiwanese classrooms.

Developing good mathematics instruction in Taiwan is motivated by several factors. The various contests of mathematics instruction have been a driving force for teachers to identify and improve the quality of mathematics teaching. Demonstrating good practices of mathematics instruction is also an efficient strategy for promoting high-quality classroom instruction valued in Taiwan. However, it is difficult to find multiple good models of mathematics teaching within a single school. Fortunately, master teachers trained by the Ministry of Education or various teacher professional development programs in Taiwan can provide a pool of good models of mathematics instruction.

While it is generally recommended to develop a learner-centered classroom instruction in Taiwan, the data reported in the TIMSS 2003 study (Mullis, et al. 2004) indicates that this approach as recommended in curriculum documents is not fully implemented in all mathematics classrooms. There are likely three possible reasons contributing to this situation. First, it has been a great challenge for teachers to change their instructional practices if they are used to teaching with a teacher-centered approach. In fact, curriculum documents only provide some general recommendations for classroom instruction with a focus on students as learners, but not prescription of what teachers should do in their own classrooms. Second, the recommended classroom instruction is often supported by mathematics educators but not mathematicians. The differentiated views toward the recommended classroom instruction suggest the need to validate and demonstrate the value of innovative instructional practices. Finally, classroom instruction as recommended in the newly revised curriculum (Ministry of Education, 2003) has not yet provided a coherent picture for what teachers need to do. In fact, there is so far no solid evidence to show whether the learner-centered approach or the teacher-centered approach is better than the other in improving students' mathematics learning (National Mathematics Advisory Panel, 2008). Good mathematics instruction as valued in Taiwan has also been a moving target that is shaped and evolved along curriculum and social-cultural changes. Nevertheless, it is the dedication of teachers and various supports in place that help to drive the continuous pursuit of mathematics classroom instruction excellence in Taiwan.

Acknowledgments Special thanks go to the teachers in the professional development program for their time and sharing their instruction. We would also like to thank three reviewers and the journal editor for their thoughtful comments on prior drafts of this article.

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