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Supporting Teachers on Maintaining High-Level Instructional Tasks in Classroom by Using Research-Based Cases

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Abstract

The study examined how teachers maintained high-level cognitive demand as instructional tasks were carried out by using research-based cases. Eight teachers enrolling in a summer course participated in this study. The course provided for conceptualize teachers' knowledge of cases, for discussing video cases, and for examining how the tasks were carried out. Task analysis and classroom observations on ordering factions were main data collected in the study. The data were analyzed according to Stein et al.'s Task Analysis Guide. The use of cases conceptualized teachers' awareness of the importance of differentiating cognitive demand levels of tasks on determining students thinking.

Key words: case method, teacher education, cognitive demand, instructional tasks

Introduction

Case method can now be used in teacher education in many countries (Dolk & den Hertog, 2001; Lin, 2002). These studies focus on answering about what and how they learn from cases. These cases are performed for various purposes (Merseth, 1996). Cases can be dilemma-driven in which portray problematic situations that require problem identification and analysis. The dilemma-driven cases aim to help teachers learn how to think about the selection of one action over another. Cases can be exemplars to establish the best practice or to make the effective teaching more public (Kleinfeld, 1992). The exemplary cases aim to assist teachers to develop the skill of critical reflection on their own practice (Stein, et al., 2000). These studies show that cases help teachers becoming more reflective practitioners, since cases reflect real situations and create challenges for teachers (Barnett, 1998). However, these studies do not indicate that how cases increases teachers' awareness of different levels required in instructional tasks resulting in students' different thinking.

Task Analysis Guide for Cognitive Demand of Tasks as the Theoretical Framework of the Study

The level of thinking in which students engage determines what they will learn. For instance, tasks that require students to perform a memorized procedure lead to

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low-level thinking, while tasks that require students to make connections to meaning lead to high-level thinking. Stein and her associates (2000) differentiate four levels of cognitive demand of instructional tasks as memorization, procedures without connection, procedures with connection, and doing mathematics. They also provide task analysis guide served as a scoring rubric for each level, as shown in Table 1.

Table 1 The Task Analysis Guide (Stein, et al., 2000)

Lower-Level Demands	***Higher-Level Demands***
1. Memorization Tasks	**3. Procedures with Connections Tasks**
➢ involving reproducing previous learned facts, rules, formula, or definitions.	➢ focus students' attention on the use of procedures for the purpose of developing deeper understanding.
➢ cannot be solved using procedures because the time frame in which the task is being completed is too short to use a procedure.	➢ are represented in multiple representations and made connections among representations that help to develop meaning.
➢ are not ambiguous -such tasks is clearly and directly stated.	➢ require some degree of cognitive effort that underlie the procedures in order to develop understanding.
2. Procedures Without Connection Tasks	**4. Doing Mathematics Tasks**
➢ have little ambiguity about what needs to be done and how to do it.	➢ have no predictable pathway explicitly suggested by the task.
➢ have no connection to the meaning that underlie the procedure being used.	➢ require students to access relevant knowledge and make appropriate use of them in working through the task.
➢ require no explanation, or explanations that focus solely on describing the procedure.	➢ require students to explore and understand the nature of mathematical concepts or relationships.

Different tasks require different levels of student thinking. Although it is important to determine the level of cognitive demand of a task, it could be happened that the high-level tasks are declined into a low-level cognitive demand when they are carried out during a lesson (Stein et al., 1996), since the Mathematical Tasks Framework (Stein et al., 2000) indicates that tasks are seen as passing through three phases: First, as they appear in textbooks or as created by teachers; Next, as they are set up by teachers in the classroom; and finally, as they are carried out by students. All of these, but especially the third phase are viewed as important influences on what students actually learn. This framework indicates that selecting a high-level task does not guarantee that students would actually think in cognitively complex ways.

Stein and her associates further suggest that task-related condition, such as mathematics topic, is one of the factors of resulting in tasks declining to the low-level. They also suggest the importance of differentiating cognitive demand of tasks is a central role in selecting instructional tasks matching to objectives. Therefore, the selection of mathematics topic is considered when examining teachers on maintaining high-level cognitive demands. Moreover, previous research on fraction shows that fraction requiring complex understanding is a challenge for students to learn because it defies students' intuitions from natural numbers (Post, 1992). Thus, fraction makes it possible for teachers to design high-level cognitive demand required in instructional tasks. However, we do not warrant that teachers create high-level demand of tasks and maintain the level during the implementation phase, unless they are aware of the importance of high-level tasks in determining students' thinking.

How can a teacher educator help teachers to provide such an opportunity to students engaging in instructional tasks in complex and meaningful ways? Stein and her associates (2000) suggest that the use of cases can be a device to play this intervention. They explain that once teachers begin to view cases of various patterns of instructional tasks, they can begin to reflect on their own practice through the lens of the cognitive demands of tasks.

Thus, this study was intended to examine how teachers maintained high-level cognitive demand when the tasks were carried out by using research-based cases. Here, the research-based cases are featured as: (1) they are real teaching; (2) they are based on valid research; (3) they are likely to initiate critical discussion by users; (4) they are constructed by classroom teachers and the researcher; and (5) the instructor of each case can be invited to participate the case discussion for articulating the context of the case teaching. The tasks referred to the study are not only the problems involving in a textbook or a lesson plan, but also the classroom activities that surround the way in which those problems are set up and actually carried out by teachers and students.

METHOD

Participants

Eight teachers, enrolling in a course called "Theory and Practice of Case Method (TPCM)" in summer program at university, participated in this study. Six participants were female and two were males. T1, T2, and T3 teachers have been teaching more than 10 years. The years of teaching for T4, T5, and T6 were ranged from 5 to 10 years. The rest of two teachers (T7, T8) had less than 5 years of teaching experience.

The TPCM Course

The cases presented in a video form were integrated into the TPCM course. The weekly two 3-hour TPCM course continuing for 48 hours consists of three parts. Part I consisting of 24 hours, reading book chapters and empirical papers were to enrich teachers' knowledge about cases. Part II containing 24 hours helped teachers learn to provide students with increased opportunities for high-level thinking. Teachers were offered with five research-based cases from which we conducted in previous studies, since videotapes allowed teachers to discern exactly what was going on as students worked on a particular task. One of the five videos was an example to illustrate what a case looks like. The case is with respect to third-graders' difficulty with understanding 2 units of $\frac{1}{7}$ equal to $\frac{2}{7}$. The cases zoomed at the instructional tasks, students' various strategies, and dialogues between students and teachers.

After viewing a video case, each case was immediately discussed in small groups and then discussed in a whole class. The author was the instructor of the TPCM, the facilitator of the case discussion, and a member of those who constructed cases. Thus, the author knew very well about the background of the case. However, the author did not provide the teachers extra information such as guiding question, even though they asked about students' preconception or objectives of the lesson. These questions they asked became the central issues for case discussion. The intention of using cases was to encourage the teachers to identify how mathematical tasks differ among the levels of cognitive demand. They were asked to answer the following questions: (1) What could be the main mathematical idea in the case? (2) Which level of cognitive demand of the instruction task would you like to place in? (3) What evidence is there that students learn these ideas or that the difficulties students have in this case?

Part III containing 6 hours was to examine how the use of research-based cases improved their ability in setting up high-level instructional tasks and how the tasks were carried out. Part III were not required but optional, because some of the teachers did not teach mathematics during the school year. Only T1, T2, T4, and T7 involved in the activities of part III. Each teacher was encouraged to put what (s)he learned from the TPCM course into classroom practice in the following school year. They took turns observing each other during the school semester when the summer course was ended. The lesson of ordering fractions with unlike denominators T1 taught were observed by T2, T4, and T7, as an example to illustrate how the T1 created instructional tasks and how the tasks were actually carried out in classroom.

Data Collection

Data for this study included case analysis of the video. These analyses were audio-taped and transcribed verbally. In addition, teachers were encouraged to write weekly reflective journals as one of the assignments of the course. The reflective journal in which drew teachers' attentions to what students are actually doing instead of on teachers themselves provides a measure of how their thinking has changed as a result of case discussion. The data collected from Part III of the course also included classroom observations. Three lessons were observed from four teachers who taught mathematics during the year. The classroom observations were videotaped and audio-taped and transcribed verbally.

This study conducted within-case and cross-case analyses to examine how the teachers learned about the cognitive demands from video research-based cases carried out in classrooms. Cross-case analyses were conducted to identify the similarities across cases and the differences among them, and overall patterns.

RESULT

The results of the study includes teachers' preconceptions of the level of cognitive domain through analyzing a task in case discussions and the effect of using cases on maintaining high-level cognitive demands in which students engaged.

Teachers' preconceptions of the level of cognitive domain through analyzing a task in case discussion

The main mathematical ideas the teachers responded to the case included constructing the meaning of fraction (CM), translation between representations of fraction (TR), and linking between iteration of unit fraction (e.g., 2 units of $\frac{1}{7}$) and fraction in part-whole model (e.g., 2 parts of seven equal-size parts) (LR).

With respect to the level of the fraction task, they put the tasks at the level of doing mathematics. The four teachers (T1, T2, T4, &T7) in one group described that the task was featured as follows. (1) It requires an explanation. (2) It is not textbook-like. (3) It involves multiple representations including the transforms from verbal to manipulatives and to diagram. (4) There is no predicable pathway suggested by the task. (5) It requires complex thinking. The other four teachers (T3, T5, T6, T8) in one group added two more features. The task activated students' misconception as $\frac{1}{7}+\frac{1}{7}=\frac{2}{14}$ and made connections between iteration of unit fraction and a non-unit fraction with part-whole model. Table 2 summarizes the responses the teachers from two different groups answered to the case.

Table 2 Responses Each Teacher Answered to Each Question

Teachers	Main Ideas	Level of cognitive demand	Features	Students Learned & Difficulties
T1 T2 T4 T7	CM TR LR	Doing mathematics	◦ requires an explanation. ◦ involves multiple representations ◦ is not textbook-like. ◦ no predicable pathway. ◦ requires complex thinking.	◦ shaded in separate areas ◦ perceptual distractors ◦ partitioning odd parts for students

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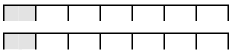
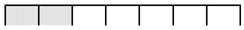
T3 T5 T6 T8	CM TR	Doing mathematics	◦ activates students' misconception ◦ makes connections between fraction meanings	◦ shaded in separate areas ◦ perceptual distractors

The teachers stated that they have learned from the tasks in the video case on provoking students' difficulties and misconceptions. They were surprised to find students' difficulty in deciding $\frac{2}{7}$ while the partitioning line in the diagram was missing. The missing line represented a perceptual distractor for students. T1 reflected on her mathematics journal and stated that

"…It is hard for the third graders mentally "put in" the line. And I was shocked with students' difficulty with agreement to "2 units of $\frac{1}{7}$ equal to $\frac{2}{7}$", since the students agreed that "2 units of $\frac{1}{7}$ is represented as , rather than represented as  (TI, Journal).

Effect of Using Cases on Maintaining High-Level Tasks

Teachers' responses to the tasks T1 set up

This phase included T1's communication to her students regarding they were expected to decide which of the fractions is greater and how they were expected to compare them. Each student was told to start to work on it and wrote individual solution on each whiteboard. T1 created the sequences tasks including comparing each of the four pairs of fraction for six graders. The four pairs ($\frac{1}{5}$ *vs.* $\frac{1}{7}$; $\frac{5}{16}$ *vs.* $\frac{5}{9}$; $\frac{4}{9}$ *vs.* $\frac{8}{12}$; $\frac{11}{12}$ *vs.* $\frac{14}{15}$) were unit fractions, fractions with same numerator or like denominator, and fractions with different numerators and denominators.

Both T2 and T4 identified the tasks T1 created as the level of "procedure with connection", while T7 identified them as low-level demands. T7 claimed that the four pairs of fractions were not involved in problem contexts and focused on producing correct answers rather than developing understanding. Conversely, T4 suggested the purposeful change on the numerals of numerator and denominator TI made on the tasks from the textbook for developing students' multiple strategies, so that these tasks required the cognitive demands of procedure with connection. T2 added for TI on deviating from the emphasis of the algorithm. T2 also suggested that the tasks were sequenced as a scaffold for developing the diversity of solutions. The patterns of the teachers' reposes to the tasks T1 created from setup phase to the implementation phase are summarized as Table 3.

Teachers' responses to the tasks T1 Implementing in classroom

The implementation phase starts as soon as students began to work on the task and continued until T1 and her students turned their attention to a new task. All three teachers T2, T4 and T7 agreed the tasks standing at the cognitive level of "procedure with connection", since six different strategies were used to solve the four types of

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fractions. In addition, the three teachers explained that in the implementation phase, T1 also invited students to explain their answers.

The teachers reported that students used two strategies to compare $\frac{1}{5}$ *vs.* $\frac{1}{7}$. One was referred to unit fraction. Students realized that there is an inverse relation between the number of parts into which the whole is divided and the resulting size of each part, so that $\frac{1}{5} > \frac{1}{7}$. The other strategy was to make into a same denominator from two different denominators. For instance, finding $\frac{1}{5} = \frac{1x7}{5x7}$, $\frac{1}{7} = \frac{1x5}{7x5}$ first, then $\frac{7}{35}$ is greater than $\frac{5}{35}$.

Table 3 Patterns of Teachers' Reposes to T1's Tasks from Setup to Implementation Phase

	In Setup Phase	In Implementation Phase	Level of Demand	Features
T2	*Procedure with connections* →	*Procedure with connections*	Maintained	◦ tasks built on students' prior knowledge. ◦ involves multiple strategies. ◦ requiring an explanation.
T4	*Procedure with connections* →	*Procedure with connections*	Maintained	◦ purposely changed the tasks with different types of fraction.. ◦ requiring an explanation. ◦ connecting procedures to meaning.
T7	*Procedure without connections* →	*Procedure with connections*	Changed	◦ involves multiple strategies. ◦ scaffolding for developing students' various strategies.

It was then followed by the problem "comparing $\frac{5}{16}$ *vs.* $\frac{5}{9}$". Most of the students still used the two previous strategies, partitioning and finding a same denominator. They further developed a new strategy of using a reference point. For instance, $\frac{5}{16}$ is less than $\frac{1}{2}$ and $\frac{5}{9}$ is greater than $\frac{1}{2}$. To this problem, TI intended to get rid of the use of common denominator, since the product of 16x5 is too big to having a correct answer. Therefore she called for Su-Jing explaining her solution.

T1: How did you change the number $\frac{5}{16}$ *into* $\frac{45}{126}$ *?*

Su-Jing: I used the fraction $\frac{5}{16}$ *with denominator and numerator multiplying 9 and got the answer* $\frac{45}{126}$.

T1: Why did 16 change into 126?

Su-Jing: I made a calculation error. It should be 144.

T1: Did all of you think it is a good strategy to find the common denominator?

Students: No.

T7 stated that the third problem "Order $\frac{4}{9}$ *vs.* $\frac{8}{12}$", with different numerators and denominators, is getting harder for students. In this case, students merely either focused on the numerator or on the denominator leading to incorrect answer. As observed, T1 encouraged her students succeeding in the problem by using: 1) reference point $\frac{1}{2}$; 2) finding a common denominator, such as finding $\frac{4x4}{9x4}$ equivalent to $\frac{4}{9}$, $\frac{8x3}{12x3}$ equivalent to $\frac{8}{12}$; 3) finding a same numerator, such as finding $\frac{4}{6}$ equivalent to $\frac{8}{12}$ and then ordering $\frac{4}{6}$ and $\frac{4}{9}$; 4) finding a same numerator, such as finding $\frac{8}{18}$ equivalent to $\frac{4}{9}$ and then ordering $\frac{8}{18}$ and $\frac{8}{12}$; 5) Making them have same denominator by multiplying their least common divisor, then compare their numerators, such as $\frac{4x4}{9x4}=\frac{16}{36}$, $\frac{8x3}{12x3}=\frac{24}{36}$.

The Table 3 shows that T7 changed her view of the levels of cognitive demands when the tasks were in the setup phase moving on to the implementation phase. The teachers finally had a commitment to TI maintaining the tasks at the level of procedure with connections. These tasks during the lesson were identified by the teachers and were featured as students’ use of multiple strategies, required students’ explanation, and connected procedures to meaning.

CONCLUSION

It is concluded that the use of cases supporting teachers on increasing their awareness of the importance of differentiating levels of cognitive demand of tasks determining students thinking. The case discussion created the opportunity of raising the level of discussing among teachers toward a deeper analysis of the relationship.

Besides, the effect of using cases on teacher’s thinking about teaching, it is found that there were usually many support factors present in teachers’ classrooms. These factors including the selection of tasks that built on students’ prior knowledge, assisting students thinking by asking thought-provoking questionings that preserve task complexity, and sustaining pressure for explanation and sense-making maintained high-level of cognitive demand of the tasks evolved during a lesson. However, it could be happened that the cognitive demands of tasks declined during the implementation phase. The factors of declining the cognitive demands of the tasks in classroom could be a further analysis for the further study.

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