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SUPPORTING TEACHERS ON UNDERSTANDING STUDENTS' LEARNING BY WRITING MATHEMATICALLY

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ABSTRACT

This study was designed to support teachers on generating writing tasks to understand students' mathematical learning. Four teachers teaching in the same grade and the researcher collaboratively set up a school-based team participating in the teacher professional development program that assists the teachers in implementing mathematical writing into classroom practices. The writing tasks the teachers generated and students' responses to the tasks were the major data collected in the study. Classroom observations and routine weekly meetings were video- and audio-taped and transcribed for analysis. The supports of generating writing tasks the teachers gained from the teacher professional program and the effect of writing mathematics on helping teachers understanding students' learning were described in the paper. The writing tasks derived from daily lesson optimized the quality the teachers' teaching and students' mathematical learning with understanding.

Key words: communication, writing mathematically, writing tasks.

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INTRODUCTION

Learning with understanding is a major goal of school mathematics curriculum (MET, 2000). When students are challenged to think and reason about mathematics, they learn to communicate the results of their thinking to others orally or in writing. This indicates that communication is central to the current vision of desirable mathematics teaching.

Communication is a way of sharing ideas and clarifying understanding. It plays a key role in helping children make important connections among physical, pictorial, symbolic, verbal, and mental representations of mathematical ideas. Representation, talking, listening, writing, and reading are key communication skills. Recent years have seen increasing interest in representing, talking, listening, and reading in mathematics classroom of Taiwan. This is evident in the literatures of research on classroom discourse (Tsai, 2004). However, little attention paid to the role of writing in mathematics teaching.

The process of creating a writing community in mathematics classroom involving complex and multifaceted undertaking is a challenge for teachers (Morgan, 1998). Teachers are challenged by the interplay between the reform vision of instruction and their own experience with more traditional tasks and pedagogy. In order that teachers are able to help their students develop writing skill, the teachers must themselves have such kind of the knowledge of writing-to-learn mathematically. Thus, helping teachers toward an instruction rich in writing mathematics in classroom is likely to require needed support from collaborative communities of practice in which writing-to-learn mathematical occurs. Thus, there is a need to design a teacher professional development program to give the needed support and the new experience for teachers.

Why do we advocate the writing work in mathematics teaching? How do teachers help students to develop their writing skill? These two questions are relevant to two key aspects of writing mathematically. The former is related to the claims of writing-to-learn, while the latter is with respect to learning-to-write. Writing-to-learn mathematics refers to the benefits of the writing mathematically, while learning-to-write mathematics refers to the strategies or techniques to improve students' knowledge and competencies of writing.

With respect to writing-to-learn mathematics, Morgan (1998) reviewing the literatures of research on writing mathematically summarized as cognitive and affective benefits. In the affective benefit, writing about feelings and about beliefs about the nature of mathematics help change students' perceptions of mathematics and to move them away from the idea that mathematics is always have a unique right answer. In the cognitive benefit, writing activity helps students to organize their thoughts, to make connections between new and old learning, and to support their mathematical thinking. The literature of writing-to-learn in mathematics shows that the types of learning depend on the different types of writing tasks (Miller, 1992). For instance, describing how a problem is solved helps students clarify their thinking and develop deeper understanding. After children have solved a problem, writing about solutions to problems helps them exhibit knowledge

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of the problem's place in the real world. The writing about word problems helps students to focus on the meanings of key words and on the relations between the words and the associated equations. The diverse types of learning interweaved students' various competencies of mathematics. However, the challenge for teachers facing the problem of writing-to-learn mathematics is how to generate creative and diverse writing tasks for students improving their learning in mathematics.

Learning-to-write mathematics is concerned about how students may learn to write in ways that their teachers value. Prior to resolving the problem, an issue that the development of learning to write is natural or it should be deliberately taught should be addressed here. The spontaneous development of learning to write is suggested as the Standards (NCTM, 1989) "This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. (P. 6)". On the contrary, many researchers appear to support the views of writing that it should be taught (Graves, 1983; Wilde, 1991). Due to writing mathematically involving the connections between the complicated and multifaceted learning, it must be developed by teachers' interventions. Hence, this study takes the position of writing mathematically that teachers need to help students to learn how to write in mathematics and the teachers' generation of writing tasks need to be supported.

THEORETICAL PERSPECTIVES OF THE STUDY

Reflection and social interaction are the two elements of the theoretical perspective of the study. This study assumes that the teachers will learn new ways of teaching mathematics only if they have opportunities to reflect on how students construct knowledge and integrate it into their existing knowledge structure. Therefore, the teachers were invited to reflect on their practical experiences in weekly meetings and developing their own insights into writing mathematically through the interaction between the participants. Teachers, as learners, can change or increase their knowledge when their cognitive structures are developed through social interaction and reflection.

Students' learning to write and teachers' learning to conduct mathematical writing tasks is seen to be situated in interactions with students (Cobb & McClain, 1999). The classroom was the primary site for the teachers' learning about writing-to-learn and learning-to write in mathematics. Learning to write and learning to conduct mathematical writing tasks develops in part by social interaction. Social constructivists assert that social constructing of knowledge occurs through its negotiation and interaction with more capable peers (Vygotsky, 1978). The activities related to generating writing tasks were structured to ensure that knowledge was actively developed by the teachers rather than imposed by the researcher. Social interactions between the teachers and the researcher create more multiple perspectives of writing mathematically. The researcher as a partner of the teachers created the opportunity of professional dialogues and helped to solve the problems when the teachers encountered in implementing writing into practices.

There was a research question to be answered: How did the teachers supported by a teacher program improve their understanding of students' learning by writing mathematically?

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Method

The Teacher Professional Development Program

The study reported here was part of the first year study of a three-year research project designed to develop a teacher professional program in which supports teachers in implementing writing into classroom practice. To reach the aim, the teachers were encouraged to use journal as a technique of gathering students' thinking processes, strategies, and their understanding. Writing tasks were served as the prompts of students' journal since 1) journal writing is likely to bring to light thoughts and understanding that typical classroom tests do not elucidate; 2) we want to establish a better means of communication among students, parents, and teachers about mathematics learning taking place in classrooms; 3) we are looking for a better way to assess individual student's entire learning process by writing about mathematics.

Supporting teachers on generating writing tasks was the core of the teacher education program. The concerns of writing tasks included: (1) supporting a method of allowing students to demonstrate their strengths, (2) stimulating students to make connections for mathematical ideas, (3) generating the writing tasks for inspecting what and how students learned from daily lesson, (4) generating high quality of writing with the potential of promoting cognitive growth, and (5) generating creative and diverse writing tasks as possible.

Settings and Participants

To achieve the aim, a school-based team consisting of four third-grade teachers were set up to discuss the issue of learning-to-write which occurred in one teacher's classroom by comparing to others. The year of teaching for four teachers (Yo, Mei, Jen, and Ying) is from 5 to 16. Four teachers were selected from the same grade so that they could support each other in their efforts to effect change. Besides, same mathematical content lent itself to a focus and similar issues of writing addressed drew attention from each teacher, leading to in-depth discussions. Thus, third-grade classrooms were one context of teachers' learning to design writing tasks and to examine the effect of writing on students' learning. Participation in routine weekly meetings was the other primary context. The role of the researcher was not to provide ready-made tasks for teachers to use, but to create opportunities for the teachers to construct their own writing tasks for students.

Classroom observation was used as a means of increasing the teachers' awareness of generating writing tasks based on the lessons that we observed altogether. Following each observation period, we met together to discuss what the teachers had observed. The three-hour weekly meetings were for providing the teachers the opportunities of sharing their creative tasks with others and helping them rethink the value of tasks in gathering information about students' in-depth understanding. Teachers were encouraged to reflect on the observations and discussions by writing reflective journals. To analyze how students responded to the tasks, they required bringing them to weekly meetings. I asked each teacher to report in public in the meeting what they learned from the tasks and what information they gathered from students' responses. Analyzing each task

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together was intended to expand each teacher's perspectives of students' learning.

Data and Analysis

Data for this study was gathered through classroom observations, writing tasks, and teachers' analyses of students' responses to the tasks throughout the entire year. The weekly meetings were audio-recorded and then were transcribed. The writing tasks and students' responses to the tasks were copied as major methods of the data collected in the study. Teachers' reflective journal is an essential method for collecting writing practice.

RESULTS

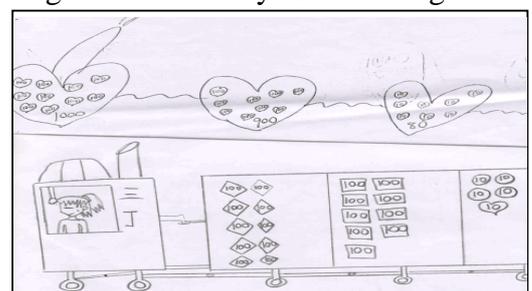
Supports for Teachers Generating Writing Tasks for Understanding Students' Learning

Initially, I would rather not provide ready-made or exemplary of writing tasks for teachers to replicate. Instead, to meet students' needs, I encouraged them to conduct personal writing tasks based on worthwhile teaching or learning events occurring in their own classrooms. In the very beginning of the study, the teachers were frustrated and helpless in generating writing tasks for understanding students learning, since the teachers, their students, and students' parents have never been encountered this new challenge. Mei expressed her frustration with the use of writing tasks as follows.

At the beginning, I asked my students to write about the key ideas that we taught today by drawings or written words. However, they did not know what should be written and their parents could not give them helps due to without their participating in classroom teaching. The number of words less than one line students wrote were related to their interesting or impressive events, such as a funny action happening in the classroom, but did not involved in mathematics learning. Accordingly, I frequently received parents' phone calls at night to ask about the activities we engaged in today's teaching. (Journal, Mei, 19990920)

Mei shared this uncomfortable experience about helping students write in mathematics in a routine meeting. Learning from Mei's experience, Yo suggested that the teachers best gave students the opportunity of learning to write in classroom since it is a new experience for students, Moreover, parents could not provide their children any help instead of their voices becoming the noise for the teachers. In the first two weeks, the writing tasks the four teachers conducted for students were only on writing about what they have learned from teaching. However, students could only write in less than ten words but also irrelevant with mathematics learning.

The first writing task involving in mathematics learning conducted by Yo was as the pilot of improving the teachers' knowledge of the generation of writing mathematically. The writing task initiated from classroom teaching was intended to clarify if the third-grade students made sense of the number sequence from 1000 to 2000. A student, Jee, responded to the task by drawing a line of the trains to represent the meanings of 1980 and 1950 as an example portrayed in



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Figure 1. The students' highly involving in teaching as we observed, Yo asked each student to write a favorite number between 1000 and 2000 in their own whiteboard. Later on, a student with the written number such as 1090 was called up as the head of the train, and then was followed by another student who had the number such as 1070 or followed the head of train if the student has the number such as 1090.

Yo brought students' responses to the routine meeting for the teachers analyzing them. After the teachers reviewing the students' work, in order to help them rethink the value of the use of writing tasks in gathering students' in-depth thinking, I asked Yo to answer the following questions: What did you expect to learn about your students from the task you conducted? Were you satisfied with your students' performance on the task? Did you really get what you want to gather? Besides, I asked each teacher to report in public what they learned from the task and what insight they gained from students' responses.

Figure 1 Jee's pictorial representation of 1980 and 1950

At the same time, the teacher Jen gave Yo's positive feedback about what she learned from students' responses. Jen found that Jee put 1000, 100, and 1 into different trunks respectively and indicated his ability of representing 1980 as 10 hundreds, 9 hundreds, and 8 tens, same as 1950. Although Jee did not learn ten hundred as one thousand yet, he has already made it true now. Therefore, most of the students experienced the relationship between thousand and hundred from this writing task.

Yo shifted her mathematics teaching with students-centered from traditional approach by connecting to students' daily life activities and by motivating students' involvement. Based on the practice, Yo also realized the type of writing task can be as the extension of the classroom teaching beyond just reviewing what happened in the classroom she did before. She stated in her reflective journal as follows.

Due to the need of generating writing task, it pushed me to figure out how to make my teaching exciting and creative. Otherwise, I did not change the way of my teaching. From the teachers' positive feedback today, I do not feel that generating writing tasks is a hard work. It seems to be likely to generate from daily lesson for the purpose of examining students' understanding. (Journal, Yo, 19991004)

The diversity and quality of writing tasks proceeded over time throughout the entire study. The writing tasks for the purpose of teachers understanding students' learning the teachers generated were sorted into the following four categories: (1) helping teachers clarify students' own mathematical thinking, (2) helping teachers developing their students' critical thinking, (3) helping teachers recognize where students need to made a remedy instruction, and (4) helping teachers understand various cognitive levels among students. Each of the tasks and students' responses to the tasks were given the examples in Lin's (2003) paper. The writing tasks for improving students' ability the teachers generated were summarized into four categories: generating story problems, problem solving, communication, and critical thinking. The categories of writing tasks of generating story problems and students' responses to the tasks were given in Lin's (2004) paper.

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The following session will focus on how the teachers created the wiring tasks for improving students' problem solving ability. The writing tasks for improving students' ability of problem solving the teachers conducted were intended to help their students to make connections what they learned in the classroom. The following two tasks were the examples to illustrate how the teachers elaborated students' learning.

Supporting Teachers Developing Students Learning with Understanding

Task 1: Composition of a 4-digit number

The task 1 as described in Figure 2, Yo conducted for clarifying students' concept of place value of 4-digit number. The teachers were impressive of the writing task, since it was a creative and attractive task in which Yo took advantage of activity of the SOGO opening celebration in the local city. Each student was excited in accomplishing this task by extended their hand to cover the money printed on the sheet as possible. The essential element of the task to be called a good task by the teachers was the key question students to be answered in the task. The question was that how many pieces of thousands, hundreds, tens, and ones you covered. This question examined if the students performed well, such as 3 thousands, 7 hundreds, 5 tens, and 8 ones turn into the number 3758.

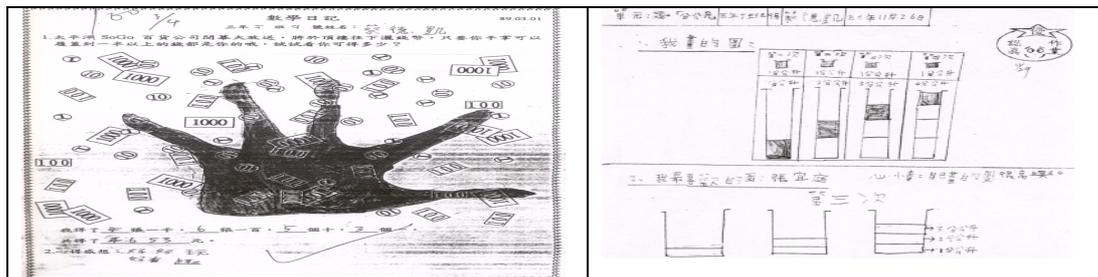


Figure 2: Task of composition of a 4-digit number

Figure 3: Task of recognizing centiliter

Task 2: Recognition of centiliter

The task 2 as depicted in Figure 3 Yo conducted was for making the connection from concrete operation to pictorial representation. In the classroom we observed, she asked each group of students to make three centiliters of water into a given container by using a cup with one-centiliter scale. After that, each group needed to draw the procedures of making three centiliters of water on their whiteboard and reported in public their performance to other groups. Following the highly interactive teaching, Yo asked students to write for summarizing what they did in the classroom. Upper of the Figure 2 indicates that the student Kei had incorrect representations. He was confused to represent between the second one centiliter and two centiliters. At the very beginning of the next day lesson, Yo asked Kei to come to the front of the class to explain his wrong answer. As a consequence, Kei corrected his wrong answer from his classmates. Besides, one of the teacher, Jen, responded to the task 2 in the week meeting and said that

I did not attend to such as Kei's misconception before. I taught this lesson without providing

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students the opportunity to operate. I just used the pictures shown on the textbook to explain to my students and asked them to practice the problems given in the students' worksheet. That's all. From Yo's sharing her students' responses to the writing task, I learned how the operation is important in learning the concept of capacity. Moreover, I found that writing task can be generated from classroom teaching and can be used as a means of clarifying students' understanding (Meeting. 19991129).

CONCLUSION AND DISCUSSION

The main conclusion of the study was that writing mathematically was an effective means of communicating students' learning and then helped the teachers understanding their students' learning. However, the effect of writing on promoting students' cognitive growth was determined by the writing tasks the teachers generated. Furthermore, in order to create creative and diverse writing tasks, it appears that the teachers needed to interact with other classroom teachers and were given the supports from the teacher professional development program.

The effect of writing mathematics on helping teachers' understanding their students learning made the correcting students' misconceptions as the common work at the beginning of the next lesson and made the teachers' mathematics teaching with highly interactive and connecting to students' daily activities. Thus, the impressive teaching and learning events occurring in classroom became the major source of conducting the writing tasks. As a result, the interactive teaching contributed the teachers to optimizing the quality of the writing tasks and instruction, and thereby optimized the students' learning.

The writing mathematically supported students' learning with understanding can be interpreted by three characteristics of learning: (1) Writing is integrative since it involves hand, eye, and brain simultaneously similar to Brunner's (1966) enactive, iconic and symbolic ways of representing reality. (2) Writing provides immediate feedback and reinforcement which are the kernel requirement for reformulation and reinforcement of learning. (3) Writing is self-pace because it connects past, present, and future through analysis and synthesis.

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