

Lin, P. J. (2003). Enhancing teachers' understanding of students' learning by using assessment tasks. *Proceedings of the 2003 Joint Meeting of 27th PME and PME-NA25 International Group for the Psychology of Mathematics Education*. Vol. 3, pp.205-212. July 11Hawaii: Honolulu.

ENHANCING TEACHERS' UNDERSTANDING OF STUDENTS' LEARNING BY USING ASSESSMENT TASKS

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Abstract

The study was designed to enhance teachers' understating of students' learning by using assessment tasks. Four third-grade teachers and the researcher collaboratively set up a school-based assessment team in the two-year Assessment Practices in Mathematics Classroom project that assisted teachers in implementing assessment as integral part of instruction. The assessment tasks along with students' responses to the tasks and classroom observation were major methods of data collection. Interweaving the tasks with analyzing students' responses to the tasks was a valuable way of assessment in which students enhanced their learning and teachers gained better understanding of students learning informing their instructional decision- making.

Introduction

There is increasing evidence that knowledge of children's thinking is a powerful influence on teachers as they consider changing their instruction (Fennema et al. 1996). A cohort of research suggests that as teachers' knowledge of students' thinking grew, their knowledge of mathematics increased, and instructional change occurred (Simon, 1995). Assessment integral to instruction contributes significantly to all students' mathematics learning (NCTM, 2000).

The Principles and Standards for School Mathematics emphasized that every instructional activity is an assessment opportunity for teacher and a learning opportunity for students (NCTM, 2000). For teachers to attain the necessary knowledge, assessment integral to instruction should become a focus in teacher education programs. Amit and Hillman (1999) provide teachers in Israel and the United States with new approaches to instruction and assessment using open-ended, real world problems. Chambers (1993) describes how teachers use the results of informal methods such as observing, listening, and questioning to help them make informed instructional decisions. The tasks involved in the previous studies on assessment were administrated merely either as a part of ongoing classroom activity for assessing students' thinking or at the end of instruction.

It is not adequate that the tasks were administrated merely during instruction to understand how each student thinks according to his natural way of thinking in a typical classroom with 30 to 35 students. Thus, there is a need to extend the use of tasks for assessing what each student knows about the material presented in each lesson. This indicates that teacher education programs should provide teachers with the opportunity to design assessment tasks, which the tasks were generated from the content of one day lesson and students' responses to the tasks were served as the decision of making further instruction of next day lesson.

The focus of this study was on helping teachers designing assessment tasks and

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analyzing students' written work, as an aspect of assessment integral to instruction. The assessment tasks were generated from the content learned in the flow of instruction. Thus, the mathematics contents included in the third-grade textbook were used as one dimension of the assessment framework of the study. Reformed curricula call for an increased emphasis on teachers' responsibility for the quality of the tasks in which students engage. The high quality of tasks should help students clarify their thinking and develop deeper understanding through formulating problems, communicating mathematics with understanding, and justifying other's way of thinking (NCTM, 2000; MET, 2000). Thus, formulating problem, communicating mathematics, justifying one another's thinking were considered as the other dimension of the assessment framework of the study.

According to De Lange (1995), a task that is open for students' process and solution is a way of stimulating students' high quality thinking. However, the design of open-ended assessment tasks is a very complex and challenging work for the teachers who are used to the traditional test. This can only be achieved by establishing an assessment team who support mutually by providing them with the opportunities for dialogue on critical assessment issues related to instruction.

The Professional Standards for Teaching Mathematics recommended that teachers could use task selection and analysis as foci for thinking about instruction and assessment (NCTM, 1991). Tasks as defined by the Standards are the problems, the questions, and exercises in which students engage (1991, p.20). Tasks referred to this study included the problems in which students engaged presented in students' journal, as an informal way of assessing what mathematics each student learned and what students' solutions displayed in a lesson. Thus, the tasks for understanding how students were thinking in the lesson were not well prepared prior to the instruction; rather, they were generated from a lesson. Analysis as defined by the Standards is the systematic reflection in which teachers engage (NCTM, 1991, p.20). Analyses referred to in the study included the reflections teachers made to monitor how well the tasks for fostering the development of every student's thinking and to examine relationships between what the teachers and their students were doing and what students learned.

The Assessment Practices in Mathematics Classroom Project

The Assessment Practices in Mathematics Classroom (APMC) project funded by the National Science Council was designed to develop a teacher program in which assisted teachers in implementing assessment into classroom practice. The paper reported here was part of the data collected in the first year study of the two-year project. An aim of the project was to assist teachers to explore their understandings about how students develop their understanding of mathematics, and how this can be supported through the program.

To reach the aim, teachers were encouraged to use students' journal as a way

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of gathering the information about students' thinking processes, strategies, and their developing mathematical understanding. Assessment tasks were served as the prompts of students' journal since 1) journal writing is likely to bring to light thoughts and understanding that typical classroom tests do not elucidate (Norwood & Carter, 1996); 2) we want to establish a better means of communication among students, parents, and teachers about mathematics learning taking place in classrooms; 3) we are looking for a better way to assess each student's entire learning process by writing about mathematics.

In generating mathematical tasks as the core of the APMC project, the concerns included that: 1) supports a method of assessment that allows students to demonstrate their strengths; 2) stimulates students to make connections for mathematical ideas; 3) promotes high quality of problem-posing, communicating, and justifying one's way of thinking; 4) generates good tasks that do not separate mathematical processes from mathematical concepts; 5) generates the assessment tasks for inspecting what and how students learned from a lesson. To generate the high quality of the tasks from today's lesson, the tasks involved in each journal including one or two problems were reasonable.

The philosophy of the APMC project was based on social constructivists' view of knowledge, in which knowledge is the product of social interaction via dialects in a professional community (Vygotsky, 1978). Therefore, activities related to generating assessment tasks were structured to ensure that knowledge was actively developed by the teachers, not imposed by the researcher. The assessment tasks in the study were generated by participants' professional dialogues and provided them with opportunities to examine their assessment practices. Thus, participants were frequently involved in observing teaching together, dialoguing as a group, and reflecting on the quality of tasks.

The study reported in this paper was designed to support teachers' understanding of students' learning by the use of assessment tasks as part of a school-based professional development project. There was a research question to be answered: How assessment tasks do teachers use in supporting their understanding of students' learning mathematics?

Methods

To achieve the goal of the study, a school-based "assessment team" consisting of the researcher and four third-grade teachers was set up to discuss the assessment issues which occurred in one teacher's classroom by comparing to others. The school involved in the project was selected because of teachers' willingness to learn, administrators' support and ever collaborating with the researcher on other projects. Four teachers were selected from same third-grade teachers so that they could support each other in their efforts to effect change. Besides, same mathematical content lent itself to a focus and similar issues of assessment

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addressed drew attention from each teacher, leading to in-depth discussions. Thus, third-grade classrooms were one context of teachers' learning to design tasks. Participation in regular weekly meetings was the other primary context. The year of teaching for four teachers (Yo, Mei, Jen, and Ying) is from 5 to 16. The role of the researcher was not to provide ready-made tasks for teachers to use, but to create opportunities for the teachers to construct their own assessment tasks for students.

The teachers had little knowledge of assessment integral to instruction, so that classroom observation was used as a means of increasing their awareness of generating assessment tasks initiated from the lessons that were observed by them together. The assessment team had routine weekly meetings for three hours. The meetings were for providing the participants with the opportunities of sharing their creative tasks with others and helping them rethink the value of tasks in gathering information about students' in-depth understanding. The participants required bring their students' responses to the tasks for others to analyze in weekly meeting. Because the use of tasks was to promote students' understanding rather than just for a work, the following questions were supplied to nudge the teachers to rethink: What do you expect to learn about your students from this task? Are you satisfied with your students' performance on the task? Did you really gain what you want to gather? Each teacher needed to report in public in the meeting what they learned from the tasks and what information they gathered from students' responses.

Data for this study was gathered through classroom observations, assessment tasks, and teachers' analyses of students' responses to the tasks throughout the entire year. The weekly meetings were audio-recorded and then were transcribed. Tasks and students' responses to the tasks were copied as major methods of the data collected in the study. Students' responses to the tasks as the examples of the paper for illustration were transcribed to be as faithful as possible to the students' exact words. Analyzing each task for teachers together was designed to expand each teacher's perspectives of students' learning.

Results

The results showed that the tasks gathering information about students' learning from students' responses to the tasks helped the teachers clarify students' own thinking, develop their students' critical thinking, recognize where students need to remediation, and understand various cognitive levels among students.

Helping teachers clarify students' own thinking mathematics

A type of task the teachers designed in the study was for helping students to clarify their own thinking. This type of task reveals that the teachers provided opportunities for their students with writing about mathematics for organizing their thinking developed in a lesson. A task shown in Figure 1 was designed to use multiple representations to illustrate mathematical ideas they learned in classroom. Ying designed the task for clarifying her students' understanding of multiplication

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by asking them to translate it into pictorial and symbolic expressions. The written work sketched in Figure 1 showed that Wei-Der clarified his concept of “equally sharing” by using pictures and symbols. The response helped the participants realized that Wei-Der did not master the basic facts of multiplication, since his answer listed a sequence of multiplication expressions from $3 \times 1 = 3$, $3 \times 2 = 6$...to $3 \times 8 = 24$ instead of getting $3 \times 8 = 24$ directly.

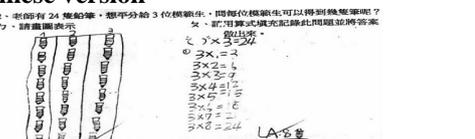
<p>Task: Chinese version</p> <p>2. 老師有 24 條鉛筆，想平均分給 3 位小朋友，每位小朋友可以得到幾支筆呢？ 請畫圖表示。</p> 	<p>Task: English version</p> <p>24 pencils Ying has were to be shared equally among 3 kids. How many pencils would each kid have? (a) Solving it by drawing. (b) Using open sentence to represent it and then solving it by using “x” operation. (Ying, 11/02/2000)</p>
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Figure 1: Wei-Der's Journal Writing

Helping teachers develop students' critical thinking

The task in Figure 2 generated from one day's lesson was administered to examine whether each student perceived others' solutions discussed in Jen's classroom. As observed, the emphasis Jen placed to how a problem is solved as important as its answer. Student's statements were open to question and elaborate from others in her classroom. The questions she asked commonly in her classroom included “Do you think it is true?” “Why do you think so?” and etc. The climate that all students of the class supported for one another's ideas was a feature of Jen's classroom (Observaiton, 03/15/2001). The following task displaying three students' methods was generated from a lesson.

<p>Task: Chinese version</p> <p>在數學習作中有一題：參加舞蹈比賽的小朋友一人發 145 公分長的緞帶，有 86 人參賽，要準備多少公分長的緞帶？</p> <p>雅惠的做法： $\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</p> <p>那菁的做法： $\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</p> <p>整瑩的做法： $\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</p> <p>你認為雅惠的做法 <input type="checkbox"/> 合理 <input type="checkbox"/> 不合理。理由： 你認為那菁的做法 <input type="checkbox"/> 合理 <input checked="" type="checkbox"/> 不合理。理由： 你認為整瑩的做法 <input type="checkbox"/> 合理 <input type="checkbox"/> 不合理。理由： 你最喜歡誰的做法？為什麼？ (Jen, 03/14/2001)</p>	<p>Task: English version</p> <p>The following three ways were used by three students for solving the problems. The problem was: <i>There are 86 kids for dance competition. Each kid needs a ribbon with 145 cm in length. How long do they need for all kids?</i></p> <p>Yei's solution: Yun's solution: Jean's solution</p> <table style="width: 100%; text-align: center;"> <tr> <td>$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</td> <td>$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</td> <td>$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$</td> </tr> </table> <p>(Jen, 03/14/2001)</p> <p>Do you agree with Yei's solution? agree disagree. Why? ___ Do you agree with Yun's solution? agree disagree. Why? ___ Do you agree with Jean's solution? agree disagree. Why? ___ Which of solutions do you like best? Why? _____</p>	$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$	$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$	$\begin{array}{r} 145 \\ \times 86 \\ \hline 870 \\ 1160 \\ \hline 12470 \end{array}$
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Figure 2: Task for Developing Students' Critical Thinking

In the task, students were asked to justify and value a method comparing to the others. This process contributes to the development of students' critical thinking. Students' responses indicated that their descriptions were not relevant with mathematical thinking, such as the algorithm used by Yei is more complex than the others'. Although the quality of students' critical thinking was not achieved as Jen's anticipation, it was a good start for students to learn to justify others' ways of thinking. This task also helped the teachers rethink what other ways could be more effective for justifying other's way of thinking.

Helping teachers understand various cognitive levels among students

The following task was administered for students who had learned the

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multiplication with one-digit number of multiplicand and multiplier. As observed, Yo paid attention to make a distinction between $6 \times 5 = ()$ and $5 \times 6 = ()$, between $0 \times 8 = ()$ and $8 \times 0 = ()$ in her lesson (Observation, 10/11/2000). However, she was not sure if her students understood the meaning of multiplicand 0 and 1. Right after the lesson, she generated a task by asking students to formulate problems represented as $1 \times 5 = ()$ and $0 \times 7 = ()$ to examine students' understanding.

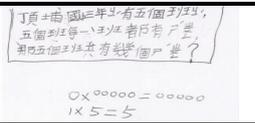
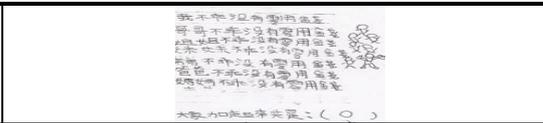
<p>Task: <i>If you were a teacher, how would you give your students a problem situation represented by the number sentence (1) $1 \times 5 = ()$ (2) $0 \times 7 = ()$. Write it in words and represent it by drawings.</i> (YO, 10/12/2000).</p>			
			
<p>English version A cow produces a bucket of milk. How many buckets of milk do 5 cows produce totally?</p>	<p>English version There are 5 third-grade classes in Din-Pu school. There are clocks in each class. How many are clocks there altogether?</p>	<p>English version: A supermarket sells eggs. There are no eggs in each carton. How many eggs did I have to pay for 7 cartons of eggs?</p>	<p>English version: I cannot get allowance. My elder brother cannot get allowance. My elder sister cannot get allowance. My younger brother cannot get allowance. My younger sister cannot get allowance. Dad cannot get allowance. Mom cannot get allowance. Because we are naughty. There is no money we get altogether.</p>

Figure 3: Sue's Writing

Figure 4: Wu's Writing

Figure 5: Liu's Writing

Figure 6: Tsai's Writing

The teachers analyzed each student's writing in a weekly meeting. They reported that 11 out of 35 students could not give a clear description of the multiplicand with number 1. Four students' responses to the task demonstrated in Figure 3, 4, 5, and 6 reveal their various cognitive levels of multiplication. Ying said that Wu had trouble with the multiplicand 1, as shown in Figure 4. He explained "1" presented in $1 \times 5 = ()$ by the words "clocks in each class" instead of "a clock in each class". The researcher recommended Yo bring the improper problem to classroom to ask other students help repair it. Next day, Yo acted as though she needed help and then asked students, "Is it [*There are 5 third-grade classes in Din-Pu school. There are clocks in each class. How many are clocks there altogether?*] wrong? Could you help me to repair it so that it can be solved?" As observed, students concentrated to repair the improper problem (Observation, 10/13/2000).

The task also showed that the third-graders had difficulty with understanding the multiplicand with number 1 by using pictorial representation. Sue drew a cow shown in Figure 3 standing for the context "each 5 cows producing a bucket milk". Likewise, Wu had difficulty with modeling the problem she posed by using $x =$ representing $1 \times 5 = 5$, as shown in Figure 4.

The teachers agreed that Tsai's cognition of multiplication, as shown in Figure 6, still stayed at "repeated addition" while Liu had better understanding of multiplication. Nevertheless, the problems Liu posed displaying in Figure 5 were not reasonable in real situation. Mei, one of the teachers, said that the cartons with no egg selling in market were not a real situation in daily life. She further suggested that the problems formulated by the students were better than those she

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had ever posed in classroom. For instance, a problem posed by another student was that “A policeman has a gun with no bullet. He fired at a robber 7 times. How many was the robber hit?” The problems students posed in students' journal became a good source for her further instruction.

Helping teachers recognize where students need to remediation

Another assessment task enabled teachers to make immediately instructional remediation for next day lesson conducted in Mei's lesson relevant with angle.

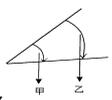
<p>Task: Chinese version 1. 小杰上课时，画了一个旋转角，并画了两个旋转方向的记号，他不知道甲和乙那一个的角度比较大，请您告诉他，并说明理由。 (Mei, 04/21/2001,</p> 	<p>Task: English version Linda drew a rotated angle with A and B as direction. She cannot tell which angle is larger in the Figure. Please help her to solve it with your explanation.</p> 	
<p>Student's response: English version They have the same size. Because both angle A and B started with the same place and ended with the same place.</p>	<p>Student's response: English version The angle B is larger than A. Because the width of angle B was longer than A, so that B was larger than A.</p>	<p>Student's response: English version The angle B is larger than A, since that the distance from the vertex to angle A is farther than to angle B.</p>

Figure 7: Shung's Writing

Figure 8: Wen's Writing

Figure 9: Chang's Writing

After grading students' journal writings, the teacher, Mei, perceived that eight of her students had a misconception of an angle. According to students' responses to the task, Mei understood that students misunderstood the size of angle either as the width between two sides of the angle, as Wen's writing in Figure 8, or as the distance between the vertex and the label of an angle, as shown Chang's writing in Figure 9. As a result, they identified the task with the incorrect answer in which B is larger than A. Mei perceived that there is a need for students to correct the misconception of an angle in next day lesson.

At the very beginning of the next day lesson, Mei asked Wen to come to the front of the classroom to explain his wrong answer. Wen said: “The width between two sides of the angle A was longer than that of the angle B”. Soon after his explanation, the class made noise, even in front of the observers. “How come?” “Impossible.” were voices of what they shouted from their seats. “Why did you disagree with Wen's thought?” Mei said. At this time, many hands were waving and Shung was pointed by the teacher to explain for Wen. Shung explained with “Both angles A and B start at the same place and stop at the same place.” displayed in Figure 7. Mei asked Shung to demonstrate what she means by “starting at the same place and stopping at the same place”. Shung exhibited an angle with a stick from a line rotated to an ending line on the blackboard (Observation, 04/23/2001).

Conclusion

The main conclusion of the study was that designing tasks along with analyzing students' responses to the task was an effective approach for enhancing teachers' knowledge of students' learning, since it provided the teachers with the opportunities to share insights of students' learning when they discussed students' responses to the tasks. The finding is consistent with the previous research on

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assessment integral to instruction (Heuvel-Panhuizen, 1996). However, the assessment tasks integral to instruction referred to in the study were characterized by the tasks conducted by the researcher collaborating with same grade classroom teachers. Most of the tasks for assessing students' learning referred to in previous studies are either designed by researchers only, or the assessment is merely a test at the end of instruction (Heuvel-Panhuizen, 1996). Comparing to assessment tasks developed by individual, sharing multiple perspectives of monitoring how well each task in a school-based assessment team was more likely to enrich the quality and the varieties of tasks. The tasks designed in the study provided more opportunities for students to clarify and extend their understanding and for teachers to gain knowledge of students' thought informing instructional decision.

A result of the study showed that the task dealing with problem posing allowed teachers to gain insights into the way students constructed mathematical understanding. The problem-posing tasks can be served as an assessment tool and gather the information of students' cognitive levels. Furthermore, the improper problems students posed as an indicator of their unclear conception can be made profitable when asking students to help repaired them and inform teachers making instructional decision. A second result in the study was that the task dealing with students' misconception seems to be likely to enable teachers to make immediately remediation for the misconceptions. Thus, correcting students' misconceptions became a common work for the teachers at the very beginning of each lesson.

Finally, the task displaying various solutions that students resolved for a problem in a lesson helped teachers examine the individual understanding to one another's' methods. The classroom discourse on mathematical ideas became a major resource of conducting such kind of assessment task. As a result, this contributed the teachers to optimizing the quality of assessment and instruction, and thereby optimized the learning of the students. The question of how effective the tasks developed in the study may be with promoting teachers' instruction will be a focus in the next stage of the study.

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